NEW CENTURY MATHS 11
MATHEMATICS STANDARD (PATHWAY 2)

FULLY WORKED SOLUTIONS

Chapter 8: Interest and depreciation

SkillCheck

Question 1

a $2.5 \div 100 = 0.025$

b $84 \div 100 = 0.84$

c $106 \div 100 = 1.06$

d $18.75 \div 100 = 0.1875$

Question 2

a $10\% \div 4 = 2.5\%$

b $7.5\% \div 2 = 3.75\%$

c $5.4\% \div 12 = 0.45\%$

Question 3

a $2653.408... \approx 2653.41$

b $12\,174.944... \approx 12\,174.94$

c $5299.091... \approx 5299.09$

Question 4

a $\frac{3}{100} \times 25\,780 = \$773.40$

b $\frac{2.5}{100} \times 85\,400 = \$2135$

c $\frac{11}{100} \times 13\,400 = \$1474$

d $\frac{4}{100} \times 8672 = \$346.88$

e $\frac{6.5}{100} \times 56\,125 = \$3648.125$
$\approx \$3648.13$

f $\frac{0.85}{100} \times 15\,600 = \$132.60$

Question 5

a $100\% + 3\% = 103\%$

$$\therefore 103\% \text{ of } \$28.50 = 1.03 \times \$28.50$$
$$= \$29.355$$
$$\approx \$29.36$$

b $100\% - 5\% = 95\%$

$$\therefore 95\% \text{ of } \$2500 = 0.95 \times \$2500$$
$$= \$2375$$

c $100\% + 10\% = 110\%$

$$\therefore 110\% \text{ of } \$88 = 1.1 \times \$88$$
$$= \$96.80$$

Question 6

a $S = V - Dn$

$$S = 75\,000 - 4300 \times 6$$
$$= 49\,200$$

b $S = V - Dn$

$$0 = V - 5200 \times 8$$
$$0 = V - 41\,600$$
$$V = 41\,600$$

c $S = V - Dn$

$$12\,000 = 52\,000 - 4800n$$
$$-40\,000 = -4800n$$
$$n = \frac{-40\,000}{-4800}$$
$$= 8.33...$$

Exercise 8.01 Percentage increase and decrease

Question 1

$\text{Total} = 200 \times \$23 = \$4600$
$100\% - 12\% = 88\%$
$\therefore \text{Total paid for calculators} = 88\% \text{ of } \$4600$
$= 0.88 \times \$4600$
$= \$4048$

Question 2

$100\% + 1.4\% = 101.4\%$
$\therefore \text{ Population} = 101.4\% \text{ of } 24\ 287\ 734$
$= 1.014 \times 24\ 287\ 734$
$= 24\ 627\ 762.2...$
$\approx 24\ 627\ 762$

Question 3

$100\% + 2.3\% = 102.3\%$
$\therefore \text{ New salary} = 102.3\% \text{ of } \$82\ 450$
$= 1.023 \times \$82\ 450$
$= \$84\ 346.35$

Question 4

$100\% - 33\% = 67\%$
$\therefore \text{ Current value} = 67\% \text{ of } \$2199$
$= 0.67 \times \$2199$
$= \$1473.33$

Question 5

$100\% + 11\% = 111\%$
$\therefore \text{ Total paid} = 111\% \text{ of } \$78.40$
$= 1.11 \times \$78.40$
$= \$87.024$
$\approx \$87.02$

Question 6

$100\% + 10\% = 110\%$

$\therefore$ Final price $= 110\%$ of $\$864$

$= 1.1 \times \$864$

$= \$950.40$

Question 7

$110\% = \$3799$

$1\% = \dfrac{\$3799}{110}$

$\therefore$ GST is $10\% = \dfrac{\$3799}{110} \times 10$

$= \$345.3636...$

$\approx \$345.36$

Question 8

$100\% - 15\% = 85\%$

$\therefore$ New price $= 85\%$ of $\$3299$

$= 0.85 \times \$3299$

$= \$2804.15$

2nd discount: $100\% - 10\% = 90\%$

$\therefore$ Amount paid $= 90\%$ of $\$2804.15$

$= 0.9 \times \$2804.15$

$= \$2523.735$

$\approx \$2523.74$

Question 9

a $100\% + 15\% = 115\%$

$\therefore$ Price after rise $= 115\%$ of $\$89$

$= 1.15 \times \$89$

$= \$102.35$

b $100\% - 8\% = 92\%$

$\therefore$ Price after discount $= 92\%$ of $\$102.35$

$= 0.92 \times \$102.35$

$= \$94.162$

$\approx \$94.16$

c Overall change in price $= \$94.16 - \$89$

$= \$5.16$ increase

$$\text{Percentage change} = \frac{\text{Overall change}}{\text{Original price}} \times 100\%$$

$$= \frac{5.16}{89} \times 100\%$$

$= 5.79...\%$

$\approx 5.8\%$ increase

Question 10

$109\% \times 104\% = 1.09 \times 1.04$

$= 1.1336$

$= 113.36\%$

$\therefore$ An increase of 13.36%

$\therefore$ D

Question 11

a Year 1: $105.2\% \times \$20\ 000 = 1.052 \times \$20\ 000 = \$21\ 040$

Year 2: $104.5\% \times \$21\ 040 = 1.045 \times \$21\ 040$

$= \$21\ 986.80$

Year 3: $104.8\% \times \$21\ 986.80 = 1.048 \times \$21\ 986.80$

$= \$23\ 042.1664$

$= \$23\ 042.17$

$\therefore$ Worth \$23 042.17 after the three years.

b Total change $= \$23\ 042.17 - \$20\ 000$

$= \$3042.17$

$$\text{Percentage change} = \frac{\text{Total change}}{\text{Original price}} \times 100\%$$

$$= \frac{3042.17}{20\ 000} \times 100\%$$

$= 15.21...\%$

$\approx$ 15.2% increase

Question 12

a Increase by 20% = 120% Decrease by 20% = 80%

$\therefore 1.2 \times 0.8 = 0.96$

$= 96\%$

$\therefore 96\% - 100\% = -4\%$

$\therefore$ Equivalent to a percentage change of 4% decrease.

b Decrease by 20% = 80% Increase by 20% = 120%

$\therefore 0.8 \times 1.2 = 0.96$

$= 96\%$

$\therefore 96\% - 100\% = -4\%$

$\therefore$ Equivalent to a percentage change of 4% decrease.

c Increase by 8% = 108%

$\therefore 1.08 \times 1.08 = 1.1664$

$= 116.64\%$

$\therefore 116.64\% - 100\% = 16.64\%$

$\therefore$ Equivalent to a percentage change of 16.64% increase.

d Increase by 10% = 110% Decrease by 15% = 85%

$\therefore 1.1 \times 0.85 = 0.935$

$= 93.5\%$

$\therefore\ 93.5\% - 100\% = -6.5\%$

$\therefore$ Equivalent to a percentage change of 6.5% decrease.

e Decrease by 4% = 96% Decrease by 6% = 94%

$\therefore\ 0.96 \times 0.94 = 0.9024$

$= 90.24\%$

$\therefore\ 90.24\% - 100\% = -9.76\%$

$\therefore$ Equivalent to a percentage change of 9.76% decrease.

f Decrease by 5% = 95% Increase by 2% = 102%

$\therefore\ 0.95 \times 1.02 = 0.969$

$= 96.9\%$

$\therefore\ 96.9\% - 100\% = -3.1\%$

$\therefore$ Equivalent to a percentage change of 3.1% decrease.

Question 13

a Year 1: $97\%\text{ of } 234\,\text{t} = 0.97 \times 234\,\text{t}$

$= 226.98\,\text{t}$

Year 2: $96\%\text{ of } 226.98\,\text{t} = 0.96 \times 226.98\,\text{t}$

$= 217.9008\,\text{t}$

Year 3: $95\%\text{ of } 217.9008\,\text{t} = 0.95 \times 217.9008\,\text{t}$

$= 207.00...\ \text{t}$

$\approx 207\,\text{t}$

$\therefore$ The mass of the iceberg after 3 years was 207 t.

b $\text{Total change} = 234\,\text{t} - 207\,\text{t} = 27\,\text{t}$

$$\text{Percentage decrease} = \frac{\text{Total decrease}}{\text{Original price}} \times 100\%$$

$$= \frac{27}{234} \times 100\%$$

$$= 11.53...\%$$

$$\approx\ 11.5\%$$

$\therefore$ The iceberg decreased by 11.5%.

Exercise 8.02 Simple interest

Question 1

$P = \$7400$, $r = 5 \div 100 = 0.05$, $n = 4$ yrs

$$\begin{aligned} I &= Prn \\ &= \$7400 \times 0.05 \times 4 \\ &= \$1480 \end{aligned}$$

$\therefore$ **C**

Question 2

a $P = \$2136$, $r = 6 \div 100 = 0.06$, $n = 2\frac{1}{2}$ yrs

$$\begin{aligned} I &= Prn \\ &= \$2136 \times 0.06 \times 2\frac{1}{2} \\ &= \$320.40 \end{aligned}$$

b $P = \$16\ 200$, $r = 7.65 \div 100 = 0.0765$, $n = 3$ yrs

$$\begin{aligned} I &= Prn \\ &= \$16\ 200 \times 0.0765 \times 3 \\ &= \$3717.90 \end{aligned}$$

c $P = \$12\ 874$, $r = 0.61 \div 100 = 0.0061$, $n = 6$ (months)

$$\begin{aligned} I &= Prn \\ &= \$12\ 874 \times 0.0061 \times 6 \\ &= \$471.1884 \\ &\approx \$471.18 \quad \text{(Round down.)} \end{aligned}$$

d $P = \$4510$, $r = 0.0301 \div 100 = 0.000301$, $n = 31$ days

$$\begin{aligned} I &= Prn \\ &= \$4510 \times 0.000301 \times 31 \\ &= \$42.08281 \\ &\approx \$42.08 \end{aligned}$$

e $P = \$20\,016$, $r = \dfrac{0.075}{12} = 0.006\,25$ (per month), $n = 5$ months

$$\begin{aligned} I &= Prn \\ &= \$20\,016 \times 0.006\,25 \times 5 \\ &= \$625.50 \end{aligned}$$

f $P = \$8250$, $r = \dfrac{0.105}{365}$ (per day), $n = 240$ days

$$\begin{aligned} I &= Prn \\ &= \$8250 \times \frac{0.105}{365} \times 240 \\ &= \$569.5890... \\ &\approx \$569.58 \quad \text{(Round down.)} \end{aligned}$$

Question 3

$I = \$262.44$, $r = 5.4 \div 100 = 0.054$, $n = 3$ yrs

$$\begin{aligned} I &= Prn \\ \$262.44 &= P \times 0.054 \times 3 \\ \$262.44 &= 0.162P \\ P &= \$262.44 \div 0.162 \\ &= \$1620 \end{aligned}$$

$\therefore$ Kalena invested \$1620.

Question 4

$I = \$5409.25$, $P = \$9835$, $n = 5$ yrs

$$\begin{aligned} I &= Prn \\ \$5409.25 &= 9835 \times r \times 5 \\ \$262.44 &= 49\,175r \\ r &= \$262.44 \div 49\,175 \\ &= 0.11 \\ &= 0.11 \times 100\% \\ &= 11\% \end{aligned}$$

$\therefore$ The interest rate was 11% per annum.

Question 5

P = \$15 750, r = 9.8 ÷ 100 = 0.098

Interest (I) = \$18 837 – \$15 750 = \$3087

$$
\begin{aligned}
I &= Prn \\
\$3087 &= \$15\ 750 \times 0.098 \times n \\
\$3087 &= \$1543.50n \\
n &= \$3087 \div \$1543.50 \\
n &= 2
\end{aligned}
$$

∴ It would take two years.

Question 6

I = \$675, P = \$7500, n = 9 ÷ 12 = 0.75 yrs

$$
\begin{aligned}
I &= Prn \\
\$675 &= \$7500 \times r \times 0.75 \\
\$675 &= 5625r \\
r &= \$675 \div 5625 \\
&= 0.12 \\
&= 12\%
\end{aligned}
$$

∴ The interest rate was 12% per annum.

Alternatively, you could make n = 9 months, giving r = 0.01 = 1% per month = 1 × 12 = 12 % per year (p.a.)

Since your answer will be per year, it is easier to work in years, making n = 0.75 years.

Question 7

I = \$3729.60, r = 16.8 ÷ 100 = 0.168, n = 3 yrs

$$
\begin{aligned}
I &= Prn \\
\$3729.60 &= P \times 0.168 \times 3 \\
\$3729.60 &= 0.504P \\
P &= \$3729.60 \div 0.504 \\
&= \$7400
\end{aligned}
$$

∴ A principal of \$7400.

Question 8

I = \$1247.36, P = \$10 460, r = 5.3 ÷ 100 = 0.053

$$I = Prn$$
$$\$1247.36 = 10\ 460 \times 0.053 \times n$$
$$\$1247.36 = 554.38 \times n$$

Now substitute the values of n in to see which gives the correct interest.

Try $n = 2$, $554.38 \times 2 = 1108.76$ --> incorrect

$n = 2.2$, $554.38 \times 2.2 = 1219.636$ --> incorrect

$n = 2.25$, $554.38 \times 2.25 = 1247.355$ --> correct

$\therefore$ C

Question 9

a For 1–3 years, r = 10.5% = 10.5 ÷ 100 = 0.105

P = \$6300, r = 0.105, n = 3 yrs

$$I = Prn$$
$$= \$6300 \times 0.105 \times 3$$
$$= \$1984.50$$

b For 7–11 months, r = 9.25% = 9.25 ÷ 100 = 0.0925

P = \$13 750, $r = \frac{0.0925}{12}$ (per month), n = 8 mths

$$I = Prn$$
$$= \$13\ 750 \times \frac{0.0925}{12} \times 8$$
$$= \$847.9166...$$
$$\approx \$847.91 \quad \text{(Round down.)}$$

c For 4–5 years, r = 12% = 12÷ 100 = 0.12

P = \$7800, r = 0.12, n = 5 (years)

$$I = Prn$$
$$= \$7800 \times 0.12 \times 5$$
$$= \$4680$$

d For 1–6 months, r = 8.75% = 8.75 ÷ 100 = 0.0875

P = \$14 240, $r = \frac{0.0875}{12}$ (per month), n = 2 mths

$$I = Prn$$
$$= \$14\ 240 \times \frac{0.0875}{12} \times 2$$
$$= \$207.666...$$
$$\approx \$207.66 \quad \text{(Round down.)}$$

Question 10

P = \$2530, n = 91 (days), I = \$80.58

$$I = Prn$$
$$\$80.58 = \$2530 \times r \times 91$$
$$\$80.58 = 230\ 230r$$
$$r = \$80.58 \div 230\ 230$$
$$= 0.000\ 349\ 997...$$
$$\approx 0.000\ 35 \quad \text{(per day)}$$

The rate = 0.000 35 × 100% = 0.035% per day.

Note that this is not money, so it is rounded normally (as asked) and not rounded down.

Question 11

P = \$4720, r = 0.67 ÷ 100 = 0.0067 (per month)

I = \$474.36

$$I = Prn$$
$$\$474.36 = \$4720 \times 0.0067 \times n$$
$$\$474.36 = \$31.624n$$
$$n = \$474.36 \div \$31.624$$
$$n = 15 \text{ months}$$

It would take 15 months or 1 year 3 months. Both answers are correct.

Question 12

Interest (I) = \$10 751.84 – \$9020 = \$1731.84

P = \$9020. Since the answer is monthly, work in months, so n = 2 × 12 = 24 months.

$$I = Prn$$
$$\$1731.84 = \$9020 \times r \times 24$$
$$\$1731.84 = 216\ 480r$$
$$r = \$1731.84 \div 216\ 480$$
$$= 0.008 \quad \text{(per month)}$$

The rate = 0.008 × 100% = 0.8% per month.

Exercise 8.03 Simple interest graphs

Question 1

Find 4 (years) on the horizontal axis and move up to the line. Move across left to find $4800 on the vertical axis. Note that each small vertical division goes up by $200.
$\therefore$ **C**.

Question 2

a Find 9 (years) on the horizontal axis and move up to the line. Move across left to find $10 800 on the vertical axis. Note that each small vertical division goes up by $200.

b Find 2.5 (years) on the horizontal axis and move up to the line. Move across left to find $3000 on the vertical axis.

c Find 7.6 (years) on the horizontal axis. Each small horizontal division represents 0.2 years, so 3 small divisions to the right of 7 years. Move up to the graph, then across left to the vertical axis. Note that each small vertical division goes up by $200. It is more than halfway between divisions, so a little more than $9100. You could say about $9100. The exact answer is $9120.

Question 3

a Find $10 000 on the vertical axis and move across right to the line.
Move down to find 8.3 years on the horizontal axis.
Note that each small horizontal division represents 0.2 years.

b Find $4200 on the vertical axis and move across right to the line.
Move down to find 3.5 years on the horizontal axis.
Note that each small horizontal division represents 0.2 years.

c Find $8500 on the vertical axis and move across right to the line.
Move down to the horizontal axis. Note that each small horizontal division represents 0.2 years.
It is less than halfway between divisions, so a little under 7.1 years.
You could say about 7.1 years. The exact answer is 7.08333… years.

Question 4

a $r = 4 \div 100 = 0.04$
$I = Prn$
$I = 4000 \times 0.04 \times n$
$I = 160n$

Construct a table including (0, 0) and using the maximum value of n, 10 years.

No. of years, *n*	0	5	10
Interest, *I* ($)	0	800	1600

Label the axes appropriately and draw the graph.

Note that any other middle point may be chosen, but (5, 800) can be plotted accurately at the intersection of two gridlines.

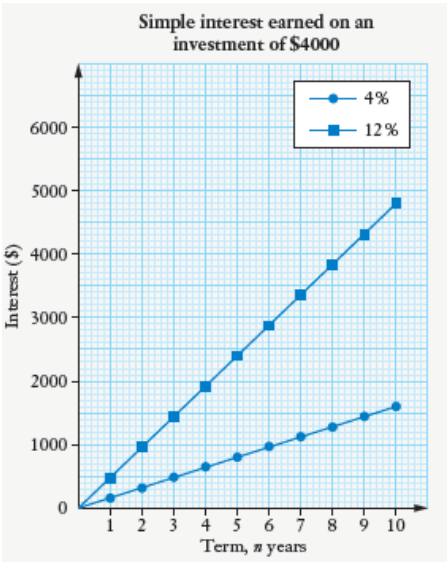
b $r = 12 \div 100 = 0.12$
$I = Prn$
$I = 4000 \times 0.12 \times n$
$I = 480n$

Construct a table including (0, 0) and using the maximum value of n, 10 years.

No. of years, *n*	0	5	10
Interest, *I* ($)	0	2400	4800

Label the axes appropriately and draw the graph.

Note that any other middle point may be chosen, but (5, 4800) can be plotted accurately at the intersection of two gridlines.



Question 5

a Use (0, 0) and (10, 4800). Other points may be chosen.

$$\frac{\text{rise}}{\text{run}} = \frac{4800 - 0}{10 - 0} = \frac{4800}{10} = 480$$

The gradient represents the simple interest earned each year.

b **i** Use the lower graph for **4a**.

Find 6 (years) on the horizontal axis and move up to the line.
Move across left to find about \$1000 on the vertical axis (or a little under \$1000).
You may say about \$1000. The exact answer is \$960.

ii Use the upper graph for **4b**.

Find 8 (years) on the horizontal axis and move up to the line.
Move across left to find about \$3800 on the vertical axis (or a little over \$3800).
You may say about \$3800. The exact answer is \$3840.

c The larger the gradient, the steeper the graph, so the second graph will be three times as steep as the first graph.

Question 6

The space between each small division on the vertical axis is \$1000 ÷ 10 = \$100.

Find \$20 000 on the vertical axis and move across right to the green line for r = 8%. Move down to find 1.5 years on the horizontal axis.

∴ **A**.

Question 7

a Find 3.5 (years) on the horizontal axis and move up to the purple line for 6%.
Move across left to find about $4200 on the vertical axis.

b Find the point for 2.5 (years) on the horizontal axis and $3000 on the vertical axis.
This point lies on the purple line for 6%, so the rate is 6%.

c Find $4500 on the vertical axis and move across right to the purple line for 6%.
Move down to find 3.75 years on the horizontal axis.

3.75 years = 3.75 × 12 months = 45 months.

d Find $7400 on the vertical axis and move across right to the green line for $r = 8\%$.
Move down to find about 4.6 years on the horizontal axis.

e 21 months in years is $21 \div 12 = 1.75$ years.

Find 1.75 years on the horizontal axis and move up to the green line for $r = 8\%$.
Move across left to find about $2800 on the vertical axis.

f 27 months in years is $27 \div 12 = 2.25$ years.

Find the point for 2.25 years on the horizontal axis and $1800 on the vertical axis.
This point lies on the red line for 4%, so the rate is 4%.

Question 8

a **i** $r = 8 \div 100 = 0.08$

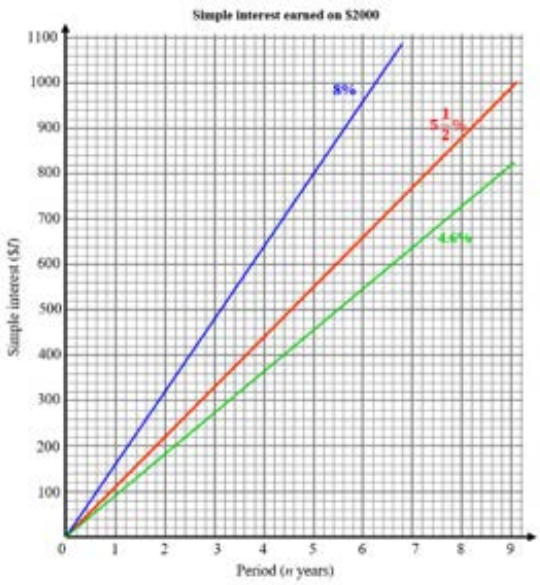
$I = Prn$

$I = \$2000 \times 0.08 \times n$

$I = 160n$

Construct a table including (0, 0) and using the maximum value of n, 9 years.

No. of years, *n*	0	5	9
Interest, *I* ($)	0	800	1440



ii $r = 4.6 \div 100 = 0.046$

$I = Prn$

$I = \$2000 \times 0.046 \times n$

$I = 92n$

Construct a table including (0, 0) and using the maximum value of n, 9 years.

No. of years, *n*	0	5	9
Interest, *I* ($)	0	460	828

b At 8%, $I = \$800$ and at 4.6%, $I = \$460$

$\therefore$ difference $= \$800 - \$460 = \$340$

$\therefore$ \$340 more interest is earned.

c **i** ≈ 4.4 years

ii ≈ 6.4 years

Question 9

You earn zero interest if money is invested for zero years.

Exercise 8.04 Compound interest

Question 1

a At the end of the first year:

$$\text{Amount} = 8000 + 5\% \times 8000 = 8000 \times 1.05$$

At the end of the second year:

$$\text{Amount} = (8000 \times 1.05) \times 1.05 = 8000 \times 1.05^2$$

At the end of the third year:

$$\text{Amount} = (8000 \times 1.05^2) \times 1.05 = 8000 \times 1.05^3$$
$$= 9261$$

$\therefore$ The investment was worth \$9261 at the end of 3 years.

b Compound interest earned = final amount – initial principal
$$= \$9261 - \$8000$$
$$= \$1261$$

c At the end of the fifth year:

$$\text{Amount} = 8000 \times 1.05^5$$
$$= 10\,210.2525$$

$\therefore$ The investment was worth \$10 210.25 at the end of 5 years.

d Increasing \$8000 by 5% *n* times:
Amount = $8000 \times (1.05)^n$

Question 2

a P = \$7400, r = 0.05, n = 4 years

$$A = P(1+r)^n$$
$$= 7400 \times (1+0.05)^4$$
$$= 8994.746...$$
$$\approx \$8994.74$$

$\therefore$ Final amount of investment is \$8994.74.

b P = \$2840, r = 0.065, n = 5 years

$$A = P(1+r)^n$$
$$= 2840 \times (1+0.065)^5$$
$$= 3891.046...$$
$$\approx \$3891.04$$

$\therefore$ Final amount of investment is \$3891.04.

c P = \$4500, r = 0.049, n = 2 years

$$A = P(1+r)^n$$
$$= 4500 \times (1+0.049)^2$$
$$= 4951.804...$$
$$\approx \$4951.80$$

$\therefore$ Final amount of investment is \$4951.80.

d P = \$17 000, r = 0.005 per month, n = 10 months

$$A = P(1+r)^n$$
$$= 17\ 000 \times (1+0.005)^{10}$$
$$= 17\ 869.382...$$
$$\approx \$17\ 869.38$$

$\therefore$ Final amount of investment is \$17 869.38.

e P = \$9250, r = 0.0082 per month, n = 6 months

$$A = P(1+r)^n$$
$$= 9250 \times (1+0.0082)^6$$
$$= 9714.532...$$
$$\approx \$9714.53$$

$\therefore$ Final amount of investment is \$9714.53.

f P = \$9000, $r = 0.084 \div 12 = 0.007$, n = 8 months

$$A = P(1+r)^n$$
$$= 9000 \times (1+0.007)^8$$
$$= 9516.522...$$
$$\approx \$9516.52$$

$\therefore$ Final amount of investment is \$9516.52.

Question 3

Inflation rate of 2.7% p.a. means that it is rising 2.7% per year every year for the 3 years. So, looking at the compound interest formula:

P = \$76 040, r = 0.027, n = 3 years

$$\begin{aligned} A &= P(1+r)^n \\ &= 76\ 040 \times (1+0.027)^3 \\ &= 82\ 367.036... \\ &\approx \$82\ 367 \end{aligned}$$

$\therefore$ Her salary in 3 years is \$82 367.

Question 4

a 10.8% p.a. = 0.108 ÷ 12 = 0.009 per month and 1 year = 12 months.

P = \$12 900, r = 0.009 per month, n = 12 months

$$\begin{aligned} A &= P(1+r)^n \\ &= 12\ 900 \times (1+0.009)^{12} \\ &= 14\ 364.274... \\ &\approx \$14\ 364.27 \end{aligned}$$

$$\begin{aligned} I &= A - P \\ &= \$14\ 364.27 - \$12\ 900 \\ &= \$1464.27 \end{aligned}$$

$\therefore$ The final amount is \$14 364.27 and the compound interest is \$1464.27.

b 7.5% p.a. = 0.072 ÷ 2 = 0.0375 per half year and 2 years = 4 half years.

P = \$13 800, r = 0.0375 per half year, n = 4 half years

$$\begin{aligned} A &= P(1+r)^n \\ &= 13\ 800 \times (1+0.0375)^4 \\ &= 15\ 989.375... \\ &\approx \$15\ 989.37 \end{aligned}$$

$$\begin{aligned} I &= A - P \\ &= \$15\ 989.37 - \$13\ 800 \\ &= \$2189.37 \end{aligned}$$

$\therefore$ The final amount is \$15 989.37 and the compound interest is \$2189.37.

c 7.5% p.a. = 0.075% ÷ 4 = 0.01625 per quarter and 2 years = 8 quarters.

P = \$13 800, r = 0.01875 per quarter, n = 8 quarters

$$\begin{aligned} A &= P(1+r)^n \\ &= 13\ 800 \times (1+0.01875)^8 \\ &= 16\ 011.059... \\ &\approx \$16\ 011.05 \end{aligned}$$

$$\begin{aligned} I &= A - P \\ &= \$16\ 011.05 - \$13\ 800 \\ &= \$2211.05 \end{aligned}$$

∴ The final amount is \$16 011.05 and the compound interest is \$2211.05.

d 9% p.a. = 0.09 ÷ 365 = $\frac{0.09}{365}$ per day.

P = \$6920, $r = \frac{0.09}{365}$ per day, n = 240 days.

$$\begin{aligned} A &= P(1+r)^n \\ &= 6920 \times \left(1 + \frac{0.09}{365}\right)^{240} \\ &= 7341.818... \\ &\approx \$7341.81 \end{aligned}$$

$$\begin{aligned} I &= A - P \\ &= \$7341.81 - \$6920 \\ &= \$421.81 \end{aligned}$$

∴ The final amount is \$7341.81 and the compound interest is \$421.81.

Question 5

A = \$24 000, r = 0.055, n = 5.

$$\$24\ 000 = P(1+0.055)^5$$
$$\$24\ 000 = P(1.055)^5$$
$$\begin{aligned} P &= \frac{\$24\ 000}{(1.055)^5} \\ &= \$18\ 363.224\ 492\ ... \\ &\approx \$18\ 363.23 \quad \text{(Round up.)} \end{aligned}$$

∴ A present value of \$18 363.23 is needed.

Question 6

a P = \$8500, r = 7% = 0.07, n = 5 years

i $I = Prn$

$= 8500 \times 0.07 \times 5$

$= \$2975$

$\therefore$ The interest earned is \$2975.

ii $A = P(1+r)^n$

$= 8500 \times (1+0.07)^5$

$= 11\ 921.689...$

$\approx \$11\ 921.68$

$I = A - P$

$= \$11\ 921.68 - \$8500$

$= \$3421.68$

$\therefore$ The interest earned is \$3421.68.

b **i** Compound interest is greater because interest is calculated on the accumulated interest as well as the principal.

ii \$3421.68 – \$2975 = \$446.68

$\therefore$ The compound interest is \$3421.68 greater.

Question 7

a P = \$21 000, r = 5% = 0.05, n = 2 years

$$\begin{aligned} A &= P(1+r)^n \\ &= 21\,000 \times (1+0.05)^2 \\ &= 23\,152.50... \\ &\approx \$23\,152.50 \end{aligned}$$

b P = \$21 000, r = 0.05 ÷ 2 = 0.025, n = 4 half years

$$\begin{aligned} A &= P(1+r)^n \\ &= 21\,000 \times (1+0.025)^4 \\ &= 23\,180.070... \\ &\approx \$23\,180.07 \end{aligned}$$

c P = \$21 000, r = 0.05 ÷ 4 = 0.0125, n = 8 quarters

$$\begin{aligned} A &= P(1+r)^n \\ &= 21\,000 \times (1+0.0125)^8 \\ &= 23\,194.208... \\ &\approx \$23\,194.20 \end{aligned}$$

d P = \$21 000, $r = \frac{0.05}{12}$, n = 24 months

$$\begin{aligned} A &= P(1+r)^n \\ &= 21\,000 \times \left(1+\frac{0.05}{12}\right)^{24} \\ &= 23\,203.768... \\ &\approx \$23\,203.76 \end{aligned}$$

e P = \$21 000, $r = \frac{0.05}{365}$, n = 2 × 365 = 730 days

$$\begin{aligned} A &= P(1+r)^n \\ &= 21\,000 \times \left(1+\frac{0.05}{365}\right)^{730} \\ &= 23\,208.430... \\ &\approx \$23\,208.43 \end{aligned}$$

Question 8

The amount of interest increases because you get interest on your interest more often.

Question 9

The following is an example. Your guess and check may be different. It is sufficient to round down to the nearest dollar.

P = \$4000, r = 4% = 0.04, A = \$4000 × 2 = \$8000.
$A = P(1 + r)^n = 4000(1 + 0.04)^n$
$= 4000(1.04)^n$

Guess, n =	**Future value** $A = 4000 \times (1.04)^n$	**Result**
25	$\$4000 \times 1.04^{25} \approx \$10\,663$	Too big
20	$\$4000 \times 1.04^{20} \approx \$8764$	Too big
15	$\$4000 \times 1.04^{15} \approx \$7203$	Too small
17	$\$4000 \times 1.04^{17} \approx \$7791$	Too small
18	$\$4000 \times 1.04^{18} \approx \$8103$	The closest

We are only looking for the nearest year, so from the table, it would take 18 years, to the nearest year.

Question 10

Using a guess and check method, then
P = \$2400, r = 8% = 0.08, A = \$3265.
$A = P(1 + r)^n = 2400(1 + 0.08)^n$
$= 2400(1.08)^n$
n = 4 gives 2400 × 1.084 = \$3265.1735 ..., so it would take 4 years.

Question 11

A = \$6260.14, r = 0.0875, n = 8.

$$\$6260.14 = P(1 + 0.0875)^8$$
$$\$6260.14 = P(1.0875)^8$$
$$P = \frac{\$6260.14}{(1.0875)^8}$$
$$= \$3199.999\,575...$$
$$\approx \$3200 \quad \text{(Round up)}$$

Given how close to \$3200 this answer is, it would be safe to say that her initial investment was \$3200.

Question 12

A = \$10 000, r = 0.0375 ÷ 4 = 0.009 375, n = 6 × 4 = 24.

$$\$10\,000 = P(1+0.009\,375)^{24}$$
$$\$10\,000 = P(1.009\,375)^{24}$$
$$P = \frac{\$10\,000}{(1.009\,375)^{24}}$$
$$= \$7993.536\,156\ldots$$
$$\approx \$7993.54 \quad \text{(Round up)}$$

Question 13

Calculate the final amount in each case.

A: P = \$10 000, r = 6% = 0.06, n = 3 years

$$I = Prn$$
$$= 10\,000 \times 0.06 \times 3$$
$$= 1800$$

Final value = \$10 000 + \$1800
= \$11 800

B: P = \$10 000, r = 5.9% = 0.059, n = 3 years

$$A = P(1+r)^n$$
$$= 10\,000 \times (1+0.059)^3$$
$$= 11\,876.483\ldots$$
$$\approx \$11\,876.48$$

C: P = \$10 000, r = 5.85% ÷ 2 = 2.925% = 0.02925, n = 3 × 2 = 6 half years

$$A = P(1+r)^n$$
$$= 10\,000 \times (1+0.02925)^6$$
$$= 11\,888.450\ldots$$
$$\approx \$11\,888.45$$

D: P = \$10 000, r = 5% ÷ 12 = $\frac{0.05}{12}$, n = 3 × 12 = 36 months

$$A = P(1+r)^n$$
$$= 10\,000 \times \left(1+\frac{0.05}{12}\right)^{36}$$
$$= 11\,614.722\ldots$$
$$\approx \$11\,614.72$$

∴ Comparing the final amounts, it is clear that option **C** is the best.

Question 14

Using a guess and check method similar to questions **7** and **8**, then

P = \$1 000 000, r = 7.4% = 0.074, A = \$2 000 000.

$A = P(1 + r)^n = 1\,000\,000(1.074)^n$

n = 9 gives \$1 901 246 and n = 10 gives \$2 041 939 (rounded down) so it would take 10 years.

Note: You can alternatively find a value of n which makes the factor $(1.074)^n$ equal to 2.

Question 15

a For a principal of \$1, when r = 5% and n = 4, the final value A = \$1.2155 (from the table).
For \$3000, A = 3000 × \$1.2155 = \$3646.50
$\therefore$ B

b For a principal of \$1, when $r = 8\%$ and $n = 5$, the final value is \$1.4693 (from the table).
When A = \$1469.30, \$1469.30 = P × \$1.4693, so P = \$1469.30 ÷ 1.4693 = \$1000,
$\therefore$ A.

c **i** For a principal of \$1, when $r = 2\%$ and $n = 3$, the final value $A = \$1.0612$ (from the table).
For \$16 000, A = 16 000 × \$1.0612 = \$16 979.20.

ii For a principal of \$1, when r = 8% and n = 6 years, the final value A = \$1.5869 (from the table).
For \$3800, A = 3800 × \$1.5869 = \$6030.22.

iii For a principal of \$1, when $r = 12\% \div 12 = 1\%$ per month and n = 5 months, the final value A = \$1.0510 (from the table).
For \$4200, A = 4200 × \$1.0510 = \$4414.20.

d For a principal of \$1, when $r = 4\% \div 2 = 2\%$ per half year and $n = 3 \times 2 = 6$ half years, A = \$1.1262 (from the table).
For \$32 000, A = 32 000 × \$1.1262 = \$36 038.40.

$$\begin{aligned} I &= A - P \\ &= \$36\,038.40 - \$32\,000 \qquad \therefore \\ &= \$4038.40 \end{aligned}$$

The interest earned is \$4038.40.

Exercise 8.05 Straight-line depreciation

Question 1

$100\% - 35\% = 65\%$

$\therefore$ 65% of $\$64\,000 = 0.65 \times 64\,000$

$= \$41\,600$

$\therefore$ A

Question 2

$100\% + 12\% = 112\%$

$\therefore$ 112% of $\$58\,600 = 1.12 \times 58\,600$

$= \$65\,632$

$\therefore$ C

Question 3

a $S = 0, \ V_0 = 11\,700, \ n = 6$

$S = V_0 - Dn$

$0 = 11\,700 - D \times 6$

$0 = 11\,700 - 6D$

$6D = 11\,700$

$D = \dfrac{11\,700}{6}$

$= \$1950$

$\therefore$ The annual depreciation is \$1950.

b **i** $V_0 = 11\,700, \ D = 1950, \ n = 1$

$S = 11\,700 - 1950 \times 1$

$= \$9750$

$\therefore$ The value is \$9750.

ii $V_0 = 11\,700, \ D = 1950, \ n = 3$

$S = 11\,700 - 1950 \times 3$

$= \$5850$

$\therefore$ The value is \$5850.

Question 4

a $S = 5000,\ V_0 = 32\,200,\ n = 5$

$$S = V_0 - Dn$$
$$5000 = 32\,200 - D \times 5$$
$$-27\,200 = -5D$$
$$D = \frac{-7200}{-5}$$
$$= \$5440$$

$\therefore$ The annual depreciation is \$5440.

b **i** $V_0 = 32\,200,\ D = 5440,\ n = 2$

$$S = 32\,200 - 5440 \times 2$$
$$= \$21\,320$$

$\therefore$ The value is \$21 230.

ii $V_0 = 32\,200,\ D = 5440,\ n = 3$

$$S = 32\,200 - 5440 \times 3$$
$$= \$15\,880$$

$\therefore$ The value is \$15 880.

Question 5

$S = 23\,000,\ V_0 = 46\,000,\ n = 3$

$$S = V_0 - Dn$$
$$23\,000 = 46\,000 - D \times 3$$
$$-23\,000 = -3D$$
$$D = \frac{-23\,000}{-3}$$
$$= \$7666.666...$$

$\therefore\ V_0 = 46\,000,\ D = 7666.666...,\ n = 6$

$$S = 46\,000 - 7666.666... \times 6$$
$$= \$0$$

$\therefore$ The value is \$0.

Question 6

a $S = 5000,\ V_0 = 180\,000,\ n = 15$

$$S = V_0 - Dn$$
$$5000 = 180\,000 - D \times 15$$
$$-175\,000 = -15D$$
$$D = \frac{-175\,000}{-15}$$
$$= \$11\,666.666...$$
$$\approx \$11\,666.67$$

$\therefore$ The annual depreciation is \$11 666.67.

b $V_0 = 180\,000,\ D = 11\,666.666...,\ n = 5$

$$S = 180\,000 - 11\,666.666... \times 5$$
$$= \$121\,666.666...$$
$$\approx \$121\,666.67$$

$\therefore$ The value is \$121 666.67.

Question 7

a $S = 35\,000,\ V_0 = 78\,000,\ n = 5$

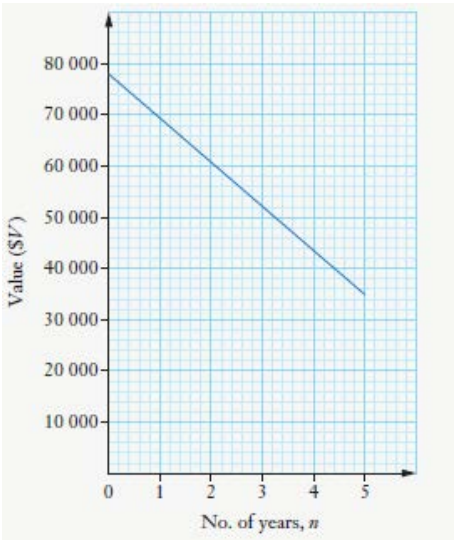
$$S = V_0 - Dn$$
$$35\,000 = 78\,000 - D \times 5$$
$$-43\,000 = -5D$$
$$D = \frac{-43\,000}{-5}$$
$$= \$8600$$

$\therefore$ The annual depreciation is \$8600.

b

No. years, *n*	0	1	2	3	4	5
Value, \$*V*	78 000	69 400	60 800	52 200	43 600	35 000

c



d \$78 000, the initial value of the van.

e \$56 000 (from graph)

f 4.4 years (from graph) OR can do algebraically:

$S = 40\,000,\ V_0 = 78\,000,\ D = 8600$

$$S = V_0 - Dn$$
$$40\,000 = 78\,000 - 8600 \times n$$
$$-38\,000 = -8600n$$
$$n = \frac{-38\,000}{-8600}$$
$$= 4.418...$$
$$\approx 4.4 \text{ years}$$

Question 8

a The value at year 0, \$5600

b **i** Value at end Yr 2 – Value at end Yr 1
$= \$3900 - \$4750$
$= -\$850$
$\therefore$ depreciation is \$850.

ii Value at end Yr 2 – Value at end Yr 0
$= \$3900 - \$5600$
$= -\$1700$
$\therefore$ depreciation is \$1700.

c **i** The value at year 2, \$3900

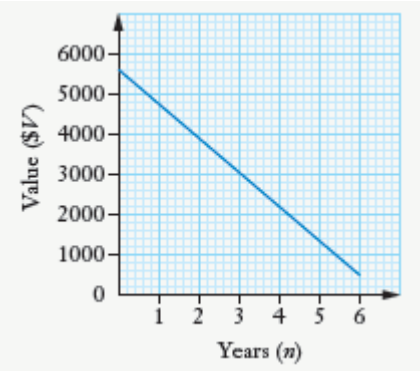
ii The value at year 4, \$2200

d

5	$3400 + 850 = 4250$	$2200 - 850 = 1350$
6	$4250 + 850 = 5100$	$1350 - 850 = 500$

Question 9

a



b \$5600, original price of scooter

c 5.4 years (from graph)

d 2.5 years (from graph)

e \$4300 (from graph)

Question 10

a The point where it crosses the vertical axis, \$19 000.

b **i** \$13 000 (from graph)

ii \$3000 (from graph)

c Value at end Yr 1 − Value at end Yr 0
$= \$17\,000 - \$19\,000$
$= -\$2000$
$\therefore$ depreciation is \$2000.

Sample HSC problem

a $P = \$15\,000$, $r = 6.5\% = 0.065$, $n = 3$ years

$$I = Prn$$
$$= \$15\,000 \times 0.065 \times 3$$
$$= \$2925$$

$\therefore$ The interest is \$2925.

b $P = \$15\,000$, $r = 6.5\% = 0.065$, $n = 3$ years

$$A = P(1+r)^n$$
$$= 15\,000 \times (1+0.065)^3$$
$$= 18\,119.244...$$
$$\approx \$18\,119.24$$
$$I = A - P$$
$$= \$18\,119.24 - \$15\,000$$
$$= \$3119.24$$

$\therefore$ The interest is \$3119.24.

c Because interest is calculated on accumulated interest as well.

Test yourself 8

Question 1

100% + 6.4% = 106.4%

$$\therefore 106.4\% \text{ of } \$72\,000 = 1.064 \times \$72\,000$$
$$= \$76\,608$$

$\therefore$ Her new salary was \$76 608.

Question 2

$$\text{Profit} = \$260 - \$145 = \$115$$

$$\%\ \text{profit} = \frac{\text{profit}}{\text{cost price}} \times 100\%$$
$$= \frac{115}{145} \times 100\%$$
$$= 79.31...\%$$
$$\approx 79.3\%$$

Question 3

$$110\% = \$1159$$

$$1\% = \frac{\$1159}{110}$$

$$\therefore \text{ GST is } 100\% = \frac{\$1159}{110} \times 100$$
$$= \$1053.636...$$
$$\approx \$1053.64$$

$\therefore$ The price of the phone before GST was \$1053.64.

Question 4

$100\% - 10\% = 90\%$

$\therefore$ After cash discount $= 90\%$ of $\$55\,000$

$= 0.9 \times \$55\,000$

$= \$49\,500$

2nd discount: $100\% - 3\% = 97\%$

$\therefore$ Amount paid $= 97\%$ of $\$49\,500$

$= 0.97 \times \$49\,500$

$= \$48\,015$

Question 5

a $100\% - 18\% = 82\%$

$\therefore$ Price after drop $= 82\%$ of $\$2400$

$= 0.82 \times \$2400$

$= \$1968$

$100\% + 5\% = 105\%$

$\therefore$ Price after rise $= 105\%$ of $\$1968$

$= 1.05 \times \$1968$

$= \$2066.40$

$\therefore$ The share price after the two changes was \$2066.40.

b Overall change $= \$2400 - \$2066.40$

$= \$333.60$ decrease

$$\text{Percentage change} = \frac{\text{Overall change}}{\text{Original price}} \times 100\%$$

$$= \frac{333.6}{2400} \times 100\%$$

$$= 13.9\%$$

$\therefore$ Overall percentage change is a decrease of 13.9%.

Question 6

a P = \$7200, r = 6.25 ÷ 100 = 0.0625, n = 3 years

$$I = Prn$$
$$= \$7200 \times 0.0625 \times 3$$
$$= \$1350$$

b P = \$4050, $r = \frac{0.03}{12} = 0.0025$ per month, n = 16 months

$$I = Prn$$
$$= \$4050 \times 0.0025 \times 16$$
$$= \$162$$

c P = \$10 300, r = 0.0075 per month, n = 2 × 12 = 24 months

$$I = Prn$$
$$= \$10\,300 \times 0.0075 \times 24$$
$$= \$1854$$

Alternatively, work in years.

P = \$10 300, r = 0.75% × 12 = 9% p.a., n = 2 (years)

$$I = Prn$$
$$= \$10\,300 \times 0.09 \times 2$$
$$= \$1854$$

d P = \$12 600, r = 0.034, $n = 3\frac{1}{2}$ years

$$I = Prn$$
$$= \$12\,600 \times 0.034 \times 3\frac{1}{2}$$
$$= \$1499.40$$

Question 7

I = \$2002.50, P = \$8900, n = 5 years

$$
\begin{aligned}
I &= Prn \\
2002.50 &= 8900 \times r \times 5 \\
2002.50 &= 44500r \\
r &= 2002.50 \div 44500 \\
&= 0.045 \\
&= 4.5\%
\end{aligned}
$$

$\therefore$ The annual interest rate was 4.5%.

Question 8

I = \$2376 P = \$4000, r = 6.6 ÷ 100 = 0.066

$$
\begin{aligned}
I &= Prn \\
2376 &= \$4000 \times 0.066 \times n \\
2376 &= 264n \\
n &= 2376 \div 264 \\
n &= 9
\end{aligned}
$$

$\therefore$ It must be invested for 9 years.

Question 9

a **i** \$4500

ii \$7300

iii \$11 300

b 8.9 years

Question 10

a $P = \$4800$, $r = 0.03$

$I = Prn$

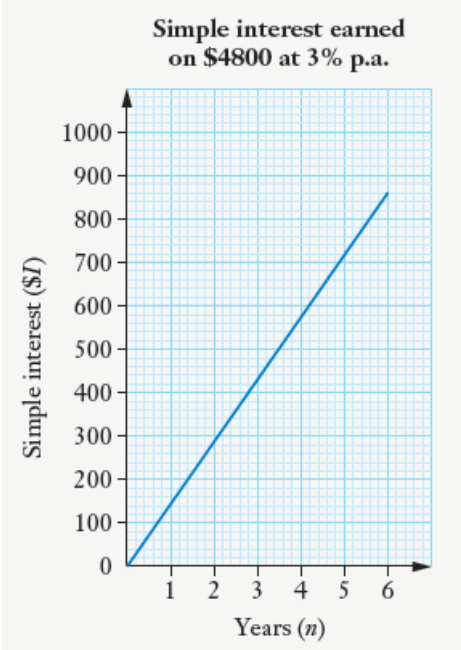
$I = \$4800 \times 0.03 \times n$

$I = 144n$

Construct a table including (0, 0) and using the maximum value of n, 6 years (any middle point can be used)

No. of years, *n*	0	5	6
Interest, *I* ($)	0	720	864

Label the axes appropriately and draw the graph.



b Use any two points.

$$m = \frac{720 - 0}{5 - 0} = \frac{720}{5} = 144$$

The gradient represents the simple interest earned each year ($144).

c Find \$500 on the vertical axis and move across right to the graph.
Move down to the horizontal axis.

$\therefore$ 3.5 years.

Question 11

a P = \$7200, r = 0.0625, n = 3 years

$$A = P(1+r)^n$$
$$= 7200 \times (1+0.0625)^3$$
$$= 8636.132...$$
$$\approx \$8636.13$$
$$I = A - P$$
$$= \$8636.13 - \$7200$$
$$= \$1436.13$$

$\therefore$ The final amount is \$8636.13 and the compound interest is \$1436.13.

b P = \$4050, r = 0.03 ÷ 12 = 0.0025 per month, n = 16 months

$$A = P(1+r)^n$$
$$= 4050 \times (1+0.0025)^{16}$$
$$= 4215.073...$$
$$\approx \$4215.07$$
$$I = A - P$$
$$= \$4215.07 - \$4050$$
$$= \$165.07$$

$\therefore$ The final amount is \$4215.07 and the compound interest is \$165.07.

c P = \$10 300, r = 0.0075 per month, n = 2 years = 24 months

$$A = P(1+r)^n$$
$$= 10\ 300 \times (1+0.0075)^{24}$$
$$= 12\ 323.059...$$
$$\approx \$12\ 323.05$$
$$I = A - P$$
$$= \$12\ 323.05 - \$10\ 300$$
$$= \$2023.05$$

$\therefore$ The final amount is \$12 323.05 and the compound interest is \$2023.05.

Question 12

A = \$15 000, r = 0.058, n = 8 (years).

$$\$15\,000 = P(1+0.058)^8$$
$$\$15\,000 = P(1.058)^8$$
$$P = \frac{\$15\,000}{(1.058)^8}$$
$$= \$9554.454\,959\ldots$$
$$\approx \$9554.46 \quad \text{(Round up.)}$$

$\therefore$ A principal of \$9554.46 is needed.

Question 13

P = \$3.60, r = 0.028, n = 4 years.

$$A = 3.6 \times (1+0.028)^4$$
$$= \$4.020\ldots$$
$$\approx \$4.02$$

$\therefore$ It will cost \$4.02.

Question 14

For half-years, r = 10% ÷ 2 = 5%

There are 4 half-years in 2 years, so n = 4.

For a principal of \$1, when r = 5% and n = 4 (half-years), the final value A = \$1.216 (from the table).

For \$9000, A = 9000 × \$1.216 = \$10 944

Interest = \$10 944 – \$9000 = \$1944

$\therefore$ The final amount is \$10 944 and the compound interest is \$1944.

Question 15

a $V_0 = 130\,000,\ D = 13\,000,\ n = 8$

$$S = V_0 - Dn$$
$$= 130\,000 - 13\,000 \times 8$$
$$= 26\,000$$

$\therefore$ The value is \$26 000.

b It depreciated at \$13 000 per annum, so it depreciated by \$13 000 in the 9th year.

Question 16

a $S = 16\,000,\ V_0 = 67\,000,\ n = 6$

$$S = V_0 - Dn$$
$$16\,000 = 67\,000 - D \times 6$$
$$-51\,000 = -6D$$
$$D = \frac{-51\,000}{-6}$$
$$= \$8500$$

$\therefore$ The yearly depreciation is \$8500.

b

Year	Accumulated depreciation ($)	Value at end of period ($S)
0	0	67 000
1	8500	58 500
2	17 000	50 000
3	25 500	44 500
4	34 000	33 000
5	42 500	24 500
6	51 000	16 000

c \$24 500

d $S = 67\,000 - 8500n$

