NEW CENTURY MATHS 11
MATHEMATICS STANDARD (PATHWAY 2)

FULLY WORKED SOLUTIONS

Chapter 5

SkillCheck

Question 1

a $20.83 \times 1000 = 20\ 830$

b $970.2 \div 10 = 97.02$

c $6.59 \times 10000 = 65\ 900$

d $72.5 \div 100 = 0.725$

e $10.4 \div 1000 = 0.0104$

f $0.0735 \times 10 = 0.735$

Question 2

a $10^7 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$
$= 10\ 000\ 000$

b $10^4 = 10 \times 10 \times 10 \times 10$
$= 10\ 000$

c $10^{-2} = \dfrac{1}{10 \times 10}$
$= \dfrac{1}{100}$
$= 0.01$

Question 3

a $A = s^2$
$= 4^2$
$= 16\ \text{cm}^2$

b $A = l \times w$
$= 15 \times 6$
$= 90\ \text{m}^2$

c $A = \pi \times r^2$
$= \pi \times 5^2$
$= 78.54\ \text{cm}^2$

d $A = \dfrac{1}{2}(a+b)h$
$= \dfrac{1}{2} \times (7+13) \times 6$
$= 60\ \text{cm}^2$

Question 4

a $A = \dfrac{1}{2}bh$
$= \dfrac{1}{2} \times 9 \times 40$
$= 180\ \text{cm}^2$

b $a^2 + b^2 = c^2$
$40^2 + 9^2 = c^2$
$1681 = c^2$
$\sqrt{1681} = c$
$41 = c$
$\therefore h = 41\ \text{cm}$

Question 5

a $250\text{ g} = \frac{1}{4}$ of a kilogram

$\therefore\ \$5.10 \div 4$

$= \$1.28$

b Cost for 5 kg = \$4.15

$\therefore$ Cost per kilogram $= \$4.15 \div 5$

$= \$0.83$

$\therefore$ Cost for three kilograms $= \$0.83 \times 3$

$= \$2.49$

Question 6

a $210\text{ cm} \div 100 = 2.1\text{ m}$

b $210\text{ cm} \times 10 = 2100\text{ mm}$

c $210\text{ cm} \div 100 = 2.1\text{ m}$

$2.1\text{ m} \div 1000 = 0.0021\text{ km}$

Question 7

a $8.1\text{ L} \div 1000 = 0.0081\text{ kL}$

b $8.1\text{ L} \times 1000 = 8100\text{ mL}$

Exercise 5.01 Metric units

Question 1

a $6.3 \text{ cm} \times 10 = 63 \text{ mm}$

b $4.36 \text{ m} \times 100 = 436 \text{ mm}$

c $7200 \text{ mm} \div 10 = 720 \text{ cm}$

$720 \text{ cm} \div 100 = 7.2 \text{ m}$

d $285 \text{ g} \div 1000 = 0.285 \text{ kg}$

e $6.9 \text{ mm} \times 1000 = 6900 \text{ μm}$

f $58\,000 \text{ mL} \div 1000 = 58 \text{ L}$

g $5.32 \text{ kg} \times 1000 = 5320 \text{ g}$

h $3400 \text{ kg} \div 1000 = 3.4 \text{ t}$

i $4720 \text{ L} \div 1000 = 4.72 \text{ kL}$

j $6000 \text{ mg} \div 1000 = 6 \text{ g}$

$6 \text{ g} \div 1000 = 0.006 \text{ kg}$

k $7.5 \times 60 = 450 \text{ min}$

$450 \text{ min} \times 60 = 27\,000 \text{ s}$

l $9.4 \times 1000 = 9400 \text{ m}$

$9400 \text{ m} \times 100 = 940\,000 \text{ cm}$

Question 2

$100 \text{ kg} \times 1000 = 100\,000 \text{g}$

$\therefore$ A

Question 3

a $169 \text{ cm} \times 10 = 1690 \text{ mm}$

b $169 \text{ cm} \div 100 = 1.69 \text{ m}$

Question 4

$57.5 \text{ kg} \times 1000 = 57\,500 \text{ g}$

Question 5

$1500 \text{ m} \div 1000 = 1.5 \text{ km}$

Question 6

a From 7.41 a.m. $\rightarrow$ 8 a.m. = 19 minutes
From 8.00 a.m. $\rightarrow$ 9 a.m. = 60 minutes
From 9.00 a.m. $\rightarrow$ 9.18 a.m. = 18minutes
$\therefore 19 + 60 + 18 = 97$ minutes

b $97 \text{ min} \times 60 = 5820$ seconds

c $97 \text{ min} \div 60 = 1.61\dot{6} \text{ h}$

$= 1\text{h and } (0.61\dot{6} \times 60) \text{ min}$
$= 1\text{h and } 37\text{min}$

Question 7

$59 \text{ kL} \times 1000 = 59\,000 \text{ L}$

Question 8

a kg

b cm or m

c m

d km

e mL or L

f mg

g cm or mm

h kL

i t

j mm or μm

Question 9

a $102 \text{ g} \div 1000 = 0.102 \text{ kg}$

b $5.2 \text{ cm} \div 100 = 0.052 \text{ m}$

Question 10

$2\,000\,000 \times 2.45\text{t} = 49\,000\,000 \text{ t}$

$49\,000\,000 \text{ t} \div 1\,000\,000 = 4.9$ megatonnes

Question 11

a $5400 \text{ m} = 5.4 \text{ km}$

$\therefore 5.4 + 3.8 + 9.5 = 18.7 \text{ km}$

b $12 + 38 + 58 = 108 \text{ min}$

$= 1 \text{ h } 48 \text{ min}$

Question 12

$8560 \text{ L} \div 1000 = 8.56 \text{ kL}$

Question 13

a 40 m or B.

b 30 000 L or D.

c 300 g or C.

d 1000 km or C.

e 12 cm or A.

f 380 mL or A.

Question 14

2.5 km = 2500m

= 250 000 cm

No. steps for dad = 250 000 ÷ 80

= 3125 steps

No. steps for Anna = 250 000 ÷ 55

= 4545.54 steps

Difference ≈ 1420

$\therefore$ Anna took approximately 1420 more steps

Question 15

$24\text{ h} \times 60 \times 60 = 86\,400\text{ s}$

Exercise 5.02 Error in measurement

Question 1

a **i** 1 m.

ii 4 m.

iii Absolute error: $\pm$ 0.5 m

Limits of accuracy: $4 \pm 0.5 = 3.5$ to 4.5 m.

b **i** 1 mm or 0.1 cm.

ii 18 mm or 1.8 cm.

iii Absolute error: $\pm$ 0.5 mm

Limits of accuracy: $18 \pm 0.5 = 17.5$ to 18.5 mm

c **i** 5 km/hr.

ii 50 km/hr.

iii Absolute error: $\pm$ 2.5 km/hr

Limits of accuracy: $50 \pm 2.5 = 47.5$ to 52.5 km/hr

d **i** 0.5°C

ii 38°C

iii Absolute error: $\pm$ 0.25°C

Limits of accuracy: $38 \pm 0.25 = 37.75$ to 38.25°C

e **i** 5 m/s.

ii 45 m/s.

iii Absolute error: $\pm$ 2.5 m/s

Limits of accuracy: $45 \pm 2.5 = 42.5$ to 47.5 m/s

f **i** 500 rpm.

ii 7000 rpm.

iii Absolute error: $\pm$ 250 rpm

Limits of accuracy: 7000 ± 250 rpm = 6750 to 7250 rpm

Question 2

When using a jug marked in millimetres, $\pm$ 0.5 mL is the absolute measurement, so C.

Question 3

The lengths that could be measured using a measuring tape with a precision of 1 cm are 37 cm and 9 cm, so A and D.

Question 4

The angle sizes that are incorrectly recorded if using a protractor marked in degrees are 103.5°, $64\frac{1}{2}°$ and 88.4°, so C, D and F.

Question 5

a ± 0.5 mL.

b 5560 mm is ± 0.5 mm.

c ± 0.05 g.

d ± 0.05 s.

e 500 grams (to the nearest gram) is ± 0.5 g.

f ± 0.5 m.

g 600 mL (to the nearest milliliter) is ± 0.5 mL.

h 500 g (to the nearest 10 grams) is ± 5 g.

i ± 0.05 cm.

j 5000 g (to the nearest 100 grams) is ± 50 g

Question 6

$$\begin{aligned}\frac{2964+3021+2938+2899}{4} &= \frac{11\,822}{4}\\ &= 2955.5\\ &\approx 2956 \text{ mm}\end{aligned}$$

Question 7

a ± 0.5 cm.

b The true measurement lies between the values of 20.5 cm and 21.5 cm.

c
$$\begin{aligned}\text{Percentage error} &= \frac{\text{absolute error}}{\text{measurement}} \times 100\%\\ &= \frac{0.5}{21} \times 100\\ &= 2.38\%\end{aligned}$$

Question 8

$$\text{Percentage error} = \frac{\text{absolute error}}{\text{measurement}} \times 100\%$$
$$= \frac{0.05}{2.3} \times 100\%$$
$$= 2.17\%$$

Question 9

$$\text{Percentage error} = \frac{\text{absolute error}}{\text{measurement}} \times 100\%$$
$$= \frac{0.5}{12\ 683} \times 100\%$$
$$= 0.0039\%$$

Question 10

a $$\text{Percentage error} = \frac{\text{absolute error}}{\text{measurement}} \times 100\%$$
$$= \frac{0.05}{15.2} \times 100\%$$
$$= 0.33\%$$

b $$\text{Percentage error} = \frac{\text{absolute error}}{\text{measurement}} \times 100\%$$
$$= \frac{5}{800} \times 100\%$$
$$= 0.625\%$$
$$\approx 0.63\%$$

c $$\text{Percentage error} = \frac{\text{absolute error}}{\text{measurement}} \times 100\%$$
$$= \frac{0.005}{0.15} \times 100\%$$
$$= 3.33\%$$

$\therefore$ 15.2 mg is the most accurate as it has the smallest percentage error.

Question 11

We don't know the absolute error; the figure could be to the nearest whole number, ten or fifty.

Question 12

38 L $\pm$ 0.5 L is correct to the nearest whole L.
38.0 L $\pm$ 0.05 L is correct to the nearest 0.1 L.
So 38.0 L is more accurate.

Question 13

Teacher to check.

Exercise 5.03 Significant figures

Question 1

The first two numbers are significant. Need to round second number up as third number is ≥ 5. Then use zero to maintain place. So, 310.
$\therefore$ B.

Question 2

a The first two numbers are significant. Then use zero to maintain place. So, 3800.

b The first two numbers are significant. Need to round second number up as third number is ≥ 5. Then use zero to maintain place. So, 2100.

c Go to first non-zero number. From there: The first two numbers are significant. Then use zero to maintain place. So, 0.0061.

d The first two numbers are significant. Need to round second number up as third number is ≥ 5. Then use zero to maintain place. So, 250 000.

e The first two numbers are significant. Need to round second number up as third number is ≥ 5. Then use zero to maintain place. So, 15 000 000.

f Go to first non-zero number. From there: The first two numbers are significant. Need to round second number up as third number is ≥ 5. Then use zero to maintain place. So, 0.000 47.

Question 3

a The first three numbers are significant. Need to round third number up as fourth number is ≥ 5. Then use zero to maintain place. So, 130.

b The first three numbers are significant. Then use zero to maintain place. So, 4980.

c Go to first non-zero number. From there: The first three numbers are significant. Then use zero to maintain place. So, 0.0106.

d The first three numbers are significant. Need to round third number up as fourth number is ≥ 5. Then use zero to maintain place. So, 1 360 000.

e The first three numbers are significant. Then use zero to maintain place. So, 25 400 000.

f Go to first non-zero number. From there: The first three numbers are significant. Then use zero to maintain place. So, 0.000 680.

Question 4

a The first number is significant. Need to round first number up as second number is ≥ 5.
Then use zero to maintain place. So, 3000.

b The first number is significant. So, 3.

c The first number is significant. Then use zero to maintain place. So, 10 000.

d Go to first non-zero number. From there: The first number is significant.
Then use zero to maintain place. So, 0.005.

e The first number is significant. Then use zero to maintain place. So, 20.

f Go to first non-zero number. From there: The first number is significant.
Need to round first number up as second number is ≥ 5.
Then use zero to maintain place. So, 0.7.

Question 5

a 9 mm is 1 sig fig $\therefore$ correct

b 7.23 mm is 3 sig fig $\therefore$ incorrect

c 24 mm is 2 sig fig $\therefore$ correct

d 6.5 mm is 2 sig fig $\therefore$ incorrect

e 31 mm is 2 sig fig $\therefore$ correct

f 10 mm could be 1 or 2 sig fig $\therefore$ correct

Question 6

a 23 μm is 2 sig fig $\therefore$ correct

b 104 μm is 3 sig fig $\therefore$ incorrect

c 19.6 μm is 3 sig fig $\therefore$ incorrect

d 832 μm is 3 sig fig $\therefore$ incorrect

e 28.6 μm is 3 sig fig $\therefore$ incorrect

f 3 μm is 1 sig fig $\therefore$ incorrect

Question 7

a $0.2 \div 0.3 = 0.666\ldots$
≈ 0.67

b $11 \div 1990 = 0.005527\ldots$
≈ 0.0055

c $16 \div 12 = 1.333\ldots$
≈ 1.3

d $\sqrt{0.0075} = 0.0866\ldots$
≈ 0.087

e $9\,300\,000 \times 0.085 = 790\,500$
$\approx 790\,000$

f $2.7^2 = 7.29$
≈ 7.3

g $\sqrt{560} = 23.66\ldots$
≈ 24

h $\sqrt{5.6} = 2.36\ldots$
≈ 2.4

i $3.4 \times 9.9 = 33.66$
≈ 34

Question 8

The first three numbers are significant. Need to round third number up as fourth number is ≥ 5.
Then use zero to maintain place. So, 7 620 000.

Question 9

a The first three numbers are significant. Need to round third number up as fourth number is ≥ 5
Then use zero to maintain place.

So, 352 000 kg.

b The first three numbers are significant. Need to round third number up as fourth number is ≥ 5.
Then use zero to maintain place. So, 4190 m.

c The first three numbers are significant. Then use zero to maintain place. So, 67.1 km/h.

d The first three numbers are significant. Need to round third number up as fourth number is ≥ 5.
Then use zero to maintain place. So, 14.8 mL.

e The first three numbers are significant. Need to round third number up as fourth number is ≥ 5.
Then use zero to maintain place.

So, 150 000 000 km.

Question 1

a Place a decimal point at the end of the number and move it 7 places to the left.
$\therefore \ 4.213 \times 10^{7}$

b Move the decimal point 2 places to the right.
$\therefore \ 1.81 \times 10^{-2}$

c Place a decimal point at the end of the number and move it 3 places to the left.
$\therefore \ 3.4 \times 10^{3}$

d Place a decimal point at the end of the number and move it 4 places to the left.
$\therefore \ 2.0 \times 10^{4}$

e Move the decimal point 3 places to the right.
$\therefore \ 3.5 \times 10^{-3}$

f Move the decimal point 4 places to the right.
$\therefore \ 2.0 \times 10^{-4}$

g Move the decimal point 1 place to the right.
$\therefore \ 3.3 \times 10^{-1}$

h Move the decimal point 3 places to the right.
$\therefore \ 4.0 \times 10^{-3}$

i Place a decimal point at the end of the number and move it 2 places to the left.
$\therefore \ 2.3 \times 10^{2}$

j Move the decimal point 5 places to the right.
$\therefore \ 7.23 \times 10^{-5}$

k Place a decimal point at the end of the number and move it 8 places to the left.
$\therefore \ 6.1 \times 10^{8}$

l Move the decimal point 8 places to the right.
$\therefore \ 8.0 \times 10^{-8}$

Question 2

a Round to two significant figures = 53 000 000
Place a decimal point at the end of the number and move it 7 places to the left.
$\therefore\ 5.3\times10^{7}$

b Round to two significant figures = 150 000
Place a decimal point at the end of the number and move it 5 places to the left.
$\therefore\ 1.5\times10^{5}$

c Round to two significant figures = 2500
Place a decimal point at the end of the number and move it 3 places to the left.
$\therefore\ 2.5\times10^{3}$

d Round to two significant figures = 0.000 46
Move the decimal point 4 places to the right.
$\therefore\ 4.6\times10^{-4}$

e Round to two significant figures = 0.0027
Move the decimal point 3 places to the right.
$\therefore\ 2.7\times10^{-3}$

f Round to two significant figures = 0.10
Move the decimal point 1 place to the right.
$\therefore\ 1.0\times10^{-1}$

g Round to two significant figures = 0.000 033
Move the decimal point 5 places to the right.
$\therefore\ 3.3\times10^{-5}$

h Round to two significant figures = 0.44
Move the decimal point 1 place to the right.
$\therefore\ 4.4\times10^{-1}$

i Round to two significant figures = 6.5
Move the decimal point 0 places to the right.
$\therefore\ 6.5\times10^{0}$

Question 3

Round to three significant figures = 357 000 000
Place a decimal point at the end of the number and move it 8 places to the left.
$\therefore\ 3.57\times10^{8}$
$\therefore$ B

Question 4

a Move the decimal point 5 places to the right and complete using place holding zeros.
$\therefore$ 740 000

b Move the decimal point 5 places to the left.
$\therefore$ 0.312

c Move the decimal point 3 places to the right and complete using place holding zeros.
$\therefore$ 1850

d Move the decimal point 4 places to the left and complete using place holding zeros.
$\therefore$ 0.000 66

e Move the decimal point 3 places to the left and complete using place holding zeros.
$\therefore$ 0.002 54

f Move the decimal point 8 places to the right and complete using place holding zeros.
$\therefore$ 475 100 000

g Move the decimal point 2 places to the left and complete using place holding zeros.
$\therefore$ 0.098

h Place a decimal point at the end of the number and move it 2 places to the right, complete using place holding zeros.
$\therefore$ 300

i Move the decimal point 2 places to the left and complete using place holding zeros.
$\therefore$ 0.054 97

j Move the decimal point 8 places to the right and complete using place holding zeros.
$\therefore$ 12 160

k Move the decimal point 1 place to the left.
$\therefore 0.802$

l Move the decimal point 3 places to the right.
$\therefore$ 6309

Question 5

a Place a decimal point at the end of the number and move it 10 places to the right.
$\therefore 1.37 \times 10^{10}$

b
$$\frac{1}{1\,000\,000} = \frac{1}{1.0 \times 10^{6}}$$
$$= 1.0 \times 10^{-6}$$

Question 6

a Place a decimal point at the end of the number.
Move the decimal point 8 places to the left and complete using place holding zeros.
$\therefore$ 0.000 000 03

b Move the decimal point 9 places to the right and complete using place holding zeros.
$\therefore$ 9 461 000 000

c Move the decimal point 6 places to the left and complete using place holding zeros.
$\therefore$ 0.000 002

d Move the decimal point 8 places to the right and complete using place holding zeros.
$\therefore$ 152 600 000

Question 7

Round to three sig fig = 7 460 000 000
Place a decimal point at the end of the number and move it 6 places to the left.
$\therefore 7.46 \times 10^{9}$

Question 8

By calculator as per example.

a 2.144×10^{7}

b 3.2×10^{5}

c 3.5×10^{4}

d 2.304×10^{-5}

e $5.314\,41 \times 10^{-10}$

f 3.76×10^{1}

g 1.26×10^{4}

h 2.3×10^{5}

Question 9

By calculator as per example.

a 3.0×10^{4}

b -3.6×10^{-3}

c 4.1×10^{12}

d 3.3×10^{-8}

e 5.9×10^{4}

f 3.4×10^{-2}

g 9.2×10^{2}

h 1.2×10^{-4}

Question 10

By calculator as per example.

a 0.0048

b 43 680 000 000

c 0.0094

d 187 690 000

e 13 640 000

f 19 063 000

g 0.000 052

h 350 000

Question 11

Answers will vary. Teacher to check.

Question 1

a Perimeter = 8 + 2 + 10 + 5 + 2 + 3
= 30 m

b Perimeter = (15 × 3) + 3 + 4 + 9 + 4 + 3
= 68 cm

c Perimeter = (15 × 3) + 12 + 5 + 4
= 66 cm

d Perimeter = 12 + 12 + 10 + 10 + 10
= 54 cm

e Perimeter = (100 × 4) + (300 × 2) + (80 × 4) + (200 × 2)
= 400 + 600 + 320 + 400
= 1720 cm

f Find the third side using Pythagoras.

$x^2 = 7^2 + 36^2$

$x^2 = 1345$

$x = \sqrt{1345}$

$x \approx 36.67$ m

$\therefore$ Perimeter = 36.67 + 7 + 36
= 79.67 m

g Find the unknown side using Pythagoras.

$x^2 = 7^2 + 5^2$

$x^2 = 74$

$x = \sqrt{74}$

$x \approx 8.6$ m

$\therefore$ Perimeter = 5 + 5 + 12 + 8.6
= 30.6 cm

h Perimeter = (8 × 6) + (13 × 4) + (36 × 2)
= 48 + 52 + 72
= 172 mm

i Perimeter = 30 + 30 + 20 + 20
= 100 cm

j Find the unknown side using Pythagoras.

$x^2 = 75^2 - 60^2$

$x^2 = 2025$

$x = \sqrt{2025}$

$x = 45$ mm

$\therefore$ Perimeter = 75 + 85 + 45 + 25
= 230 mm

k Perimeter = 10 × 4
= 40 cm

l Perimeter = 7 + 3 + 7 + 13 + 13 + 15
= 58 cm

Question 2

a Circumference = $2\pi r$
$= 2 \times \pi \times 8.5$
= 53.407...
$\approx$ 53.4 m

b Perimeter = $\frac{2\pi r}{2} + 4$
$= \frac{2 \times \pi \times 2}{2} + 4$
$\approx$ 10.3 m

OR

Perimeter = $\frac{\pi d}{2} + 4$
$= \frac{\pi \times 4}{2} + 4$
$\approx$ 10.3 m

c Perimeter = $\frac{2\pi r}{4} + 6 + 6$
$= \frac{2 \times \pi \times 6}{4} + 12$
$\approx$ 21.4 cm

d Perimeter = $\frac{2\pi r}{2} + 6 + 4 + 6$
$= \frac{2 \times \pi \times 2}{2} + 6 + 4 + 6$
$\approx$ 22.3 cm

e Perimeter = $\frac{3}{4} \times (2\pi \times 15) + 15 + 15$
$\approx$ 100.7 mm

f To find unknown straight side use Pythagoras. Sides are the hypotenuse, $15-9=6$ cm and 8 cm.

$x^2 = 6^2 + 8^2$

$x^2 = 100$

$x = \sqrt{100}$

$x = 10$ cm

Circumference of the curve $= \dfrac{2\pi r}{2}$

$= \dfrac{2\times\pi\times4}{2}$

≈ 12.6 cm

$\therefore$ Perimeter = 10 + 15 + 12.6 + 9

= 46.6 cm

g Perimeter $= \dfrac{2\pi r}{2}+10+10$

$= \dfrac{2\times\pi\times10}{2}+20$

≈ 51.4 cm

h Two semi-circles make a whole circle.

$\therefore$ Perimeter $= 2\pi r+26+26$

$= 2\times\pi\times11+52$

≈ 121.1 mm

i Perimeter $= \dfrac{2\pi r}{2}+\dfrac{2\pi r}{2}+6$

$= \dfrac{2\times\pi\times6}{2}+\dfrac{2\times\pi\times3}{2}+6$

≈ 34.3 m

j Perimeter $= \left[\dfrac{3}{4}\times(2\pi r)\right]\times4$

$= \left[\dfrac{3}{4}\times(2\times\pi\times5)\right]\times4$

≈ 94.2 cm

k Perimeter $= \left[\dfrac{40}{360}\times(2\pi r)\right]+16+16$

$= \left[\dfrac{40}{360}\times(2\times\pi\times16)\right]+32$

≈ 43.2 m

l Perimeter $= \dfrac{2\pi r}{2}+\dfrac{2\pi r}{2}+5+5$

$= \dfrac{2\times\pi\times10}{2}+\dfrac{2\times\pi\times5}{2}+10$

≈ 57.1 cm

Question 3

a **i** $l = \frac{\theta}{360} \times 2\pi r$

$= \frac{80}{360} \times 2 \times \pi \times 12$

$\approx 16.8 \text{ m}$

ii $\therefore$ Perimeter $= r + r + \frac{\theta}{360} \times 2\pi r$

$= 12 + 12 + 16.8$

$\approx 40.8 \text{ m}$

b **i** $l = \frac{\theta}{360} \times 2\pi r$

$= \frac{90}{360} \times 2 \times \pi \times 9.6$

$\approx 15.1 \text{ m}$

ii $\therefore$ Perimeter $= r + r + \frac{\theta}{360} \times 2\pi r$

$= 9.6 + 9.6 + 15.1$

$\approx 34.3 \text{ m}$

c **i** $l = \frac{\theta}{360} \times 2\pi r$

$= \frac{135}{360} \times 2 \times \pi \times 86$

$\approx 202.6 \text{ mm}$

ii $\therefore$ Perimeter $= r + r + \frac{\theta}{360} \times 2\pi r$

$= 86 + 86 + 202.6$

$\approx 374.6 \text{ mm}$

Question 4

Two semi-circles make a whole circle.

Ali: Large circle → radius of $\frac{20 + 4 + 4}{2} = 14 \text{ m}$.

$\therefore$ distance $= 2\pi r + 30 + 30$

$= 2 \times \pi \times 14 + 60$

$\approx 247.96 \text{ m}$

Billy**:** Smaller one → radius of $\frac{20}{2} = 10 \text{ m}$.

$\therefore$ distance $= 2\pi r + 80 + 80$

$= 2 \times \pi \times 10 + 160$

$\approx 222.83 \text{ m}$

$\therefore$ difference $= 247.96 \text{ m} - 222.83 \text{ m}$

$= 25.13 \text{ m}$ difference

$\therefore$ Ali ran a distance of 25.1 m more than Billy, correct to the nearest 0.1 m.

Question 5

a $\theta = 360° \div 8$
$= 45°$

b radius $= \frac{30}{2} = 15$ cm

$\therefore$ Perimeter $= r + r + \frac{\theta}{360} \times 2\pi r$
$= 15 + 15 + \frac{45}{360} \times 2 \times \pi \times 5$
$= 30 + 11.78...$
$= 41.78...$
≈ 41.8 cm

Question 1

$300 \text{ m}^2 \times 100 \times 100 = 3\,000\,000 \text{ cm}^2$
$\therefore$ C

Question 2

$5 \text{ km}^2 = 5\,000\,000 \text{ m}^2$

$\frac{5\,000\,000}{10\,000} = 500 \text{ ha}$
$\therefore$ A

Question 3

a $5 \text{ m}^2 \times 100 \times 100 = 50\,000 \text{ cm}^2$

b $2500 \text{ cm}^2 \div 10 \div 10 = 25 \text{ mm}^2$

c $72\,000 \text{ m}^2 \div 10\,000 = 7.2 \text{ ha}$

d $6800 \text{ cm}^2 \div 100 \div 100 = 0.68 \text{ m}^2$

e $3.09 \text{ km}^2 \times 1000 \times 1000 = 3\,090\,000 \text{ m}^2$

f $3.6 \text{ km}^2 \times 1000 \times 1000 = 3\,600\,000 \text{ m}^2$
$3\,600\,000 \text{ m}^2 \div 10\,000 = 360 \text{ ha}$

g $4.73 \text{ m}^2 \times 100 \times 100 \times 10 \times 10 = 4\,730\,000 \text{ mm}^2$

h $540 \text{ ha} \times 10\,000 = 5\,400\,000 \text{ m}^2$
$5\,400\,000 \text{ m}^2 \div 1000 \div 1000 = 5.4 \text{ km}^2$

Question 4

$801\,600 \text{ km}^2 \times 1000 \times 1000 = 801\,600\,000\,000 \text{ m}^2$

$801\,600\,000\,000 \text{ m}^2 \div 10\,000 = 8.016 \times 10^7 \text{ ha}$

Question 5

a $2300 \times 1880 = 4\,209\,000 \text{ mm}^2$
Convert to m^2.
$4\,209\,000 \div 10 \div 10 \div 100 \div 100 = 4.209 \text{ m}^2$

b $2300 \text{ mm} \times 1830 \text{ mm} = 2.3 \text{ m} \times 1.83 \text{ m}$
$= 4.209 \text{ m}^2$

The answers are the same.

Question 6

Area = area of rectangle + area of triangle

$= 45 \times 58 + \frac{1}{2} \times 45 \times 23$

$= 3127.5 \text{ m}^2$

$\therefore$ **A**

Question 7

$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{70}{360} \times \pi \times 8^2$

$\approx 39.1 \text{ cm}^2$

$\therefore$ C.

Question 8

a Area $= s^2$

$= 8^2$

$= 64 \text{ m}^2$

b Area $= \frac{1}{2}(a+b)h$

$= \frac{1}{2} \times (43+90) \times 65$

$= 4322.5 \text{ m}^2$

c Area $= \frac{1}{2}bh$

$= \frac{1}{2} \times 6 \times 8$

$= 24 \text{ m}^2$

d Area $= \frac{1}{2}xy$

$= \frac{1}{2} \times 10 \times 8$

$= 40 \text{ m}^2$

e Area $= b \times h$

$= 15 \times 25$

$= 375 \text{ m}^2$

i Area $= \frac{\pi r^2}{2}$

$= \frac{\pi \times 7.5^2}{2}$

$= 88.35\ldots$

$\approx 88.4 \text{ m}^2$

f Area $= \pi r^2$

$= \pi \times 1200^2$

$= 4\,523\,893.42\ldots$

$\approx 4\,523\,893.4 \text{ cm}^2$

g Area $= \frac{1}{2}(a+b)h$

$= \frac{1}{2} \times (18+32) \times 12$

$= 300 \text{ m}^2$

h To find unknown straight side use Pythagoras.

Sides are the hypotenuse 5 m and side 3 m.

$5^2 = x^2 + 3^2$

$x^2 = 5^2 - 3^2$

$x^2 = 16$

$x = \sqrt{16}$

$x = 4 \text{ m}$

Area $= \frac{1}{2}bh$

$= \frac{1}{2} \times 3 \times 4$

$= 6 \text{ m}^2$

j $\text{Area} = \pi r^2 - \pi r^2$

$= \pi \times 12^2 - \pi \times 9^2$

$= 197.92\ldots$

$\approx 197.9 \text{ cm}^2$

k $\text{Area 1} = \frac{1}{2}bh$

$= \frac{1}{2} \times 8 \times 12$

$= 48 \text{ m}^2$

$\text{Area 2} = \frac{1}{2}bh$

$= \frac{1}{2} \times 12 \times 10$

$= 60 \text{ m}^2$

$\text{Area 3} = \frac{1}{2}bh$

$= \frac{1}{2} \times 18 \times 9$

$= 81 \text{ m}^2$

Total area = 48 + 60 + 81

$= 189 \text{ m}^2$

Question 9

a $A_{\text{shaded}} = A_{\text{outer rectangle}} - A_{\text{inner rectangle}}$

$= 6 \times 20 - 9 \times 3$

$= 93 \text{ m}^2$

b $\text{Area} = bh$

$= 4 \times 9$

$= 36 \text{ cm}^2$

c $A_{\text{shaded}} = A_{\text{square}} - A_{\text{circle}}$

$= 190 \times 190 - \pi \times 95^2$

$= 7747.1263...$

$\approx 7747 \text{ mm}^2$

d $\text{Area} = A_{\text{rectangle}} + A_{\text{rectangle}} + A_{\text{rectangle}}$

$\text{Area} = 2 \times 5 + 5 \times 15 + 7 \times 6$

$= 127 \text{ m}^2$

e $A_{\text{shaded}} = A_{\text{rectangle}} - A_{\text{trapezium}}$

$= 10 \times 6 - \frac{1}{2}(2.5 + 6) \times 7$

$= 30.25$

$\approx 30 \text{ m}^2$

f $A_{\text{shaded}} = A_{\text{outer rectangle}} + A_{\text{outer circle}} - A_{\text{inner circle}}$

$= 7.8 \times 10.2 + \pi \times \left(\frac{7.8}{2}\right)^2 - \pi \times 3.5^2$

$= 88.859\,11...$

$\approx 89 \text{ m}^2$

Question 10

a
$$A = \frac{\theta}{360} \times \pi r^2$$
$$= \frac{30}{360} \times \pi \times 5^2$$
$$= 6.544...$$
$$\approx 7 \text{ m}^2$$

b
$$A = \frac{\theta}{360} \times \pi r^2$$
$$= \frac{120}{360} \times \pi \times 90^2$$
$$= 8482.30...$$
$$\approx 8482 \text{ cm}^2$$

c
$$A = \frac{\theta}{360} \times \pi r^2$$
$$= \frac{75}{360} \times \pi \times 480^2$$
$$= 150\ 796.44...$$
$$\approx 150\ 796 \text{ m}^2$$

Question 11

a
$$\text{Area} = \text{A}_{\text{outer rectangle}} - \text{A}_{\text{rectangle}} - \text{A}_{\text{rectangle}}$$
$$= 10 \times 7 - 3 \times 2 - 3 \times 4$$
$$= 52 \text{ m}^2$$

b
$$\text{Area} = \text{A}_{\text{square}} + 4 \times \text{A}_{\text{semi-circle}}$$
$$= 8 \times 8 + 4 \times \frac{1}{2} \times \pi \times 4^2$$
$$= 164.530\,96...$$
$$\approx 165 \text{ m}^2$$

c Using Pythagoras with hypotenuse of 10 cm and side of 6 cm:

$$\text{Height of triangle} = \sqrt{10^2 - 6^2}$$
$$= 8$$

$$\text{Area} = \text{A}_{\text{triangle}} + \text{A}_{\text{semi-circle}}$$
$$= \frac{1}{2} \times 8 \times 12 + \frac{1}{2} \times \pi \times 6^2$$
$$= 104.548\,66...$$
$$\approx 105 \text{ m}^2$$

d
$$\text{Area} = \text{A}_{\text{rectangle}} \text{A}_{\text{triangle}}$$
$$= 9.6 \times 2.8 + \frac{1}{2} \times 9.6 \times 1.3$$
$$= 33.12$$
$$\approx 33 \text{ m}^2$$

e
$$\text{A}_{\text{shaded}} = \text{A}_{\text{rectangle}} + \text{A}_{\text{trapezium}}$$
$$= 12 \times 8 + \frac{1}{2} \times (12 + 6) \times 8$$
$$= 168 \text{ m}^2$$

f
$$\text{Area} = \text{A}_{\text{trapezium}} + \text{A}_{\text{trapezium}}$$
$$= \frac{1}{2}(6 + 11) \times 4 + \frac{1}{2}(5 + 11) \times 4$$
$$= 66 \text{ m}^2$$

Question 12

$$\text{Area 1} = \frac{\pi r^2}{2}$$
$$= \frac{\pi \times 2^2}{2}$$
$$\approx 6.3 \text{ m}^2$$

$$\text{Area 2} = l \times w$$
$$= 4 \times 9$$
$$= 36 \text{ m}^2$$

$$\therefore \text{ Total area} = 6.3 + 36$$
$$= 42.3 \text{ m}^2$$

Question 13

a $$\text{Area 1} = l \times w$$
$$= 200 \times 60$$
$$= 12\,000 \text{ m}^2$$

$$\text{Area 2} = \pi r^2$$
$$= \pi \times 30^2$$
$$= 2827.433\ldots \text{ m}^2$$

$$\text{Total area} = 12\,000 + 2827.433\ldots$$
$$= 14\,827.433\ldots \text{ m}^2$$
$$\approx 15\,000 \text{ m}^2 \text{ correct to 2 sig figs}$$

b $$\text{Radius} = \frac{60}{2} = 30 \text{ m}$$

$$\therefore \text{Perimeter} = 200 + 200 + 2 \times (\frac{1}{2} \times 2\pi r)$$
$$= 400 + 2 \times (\frac{1}{2} \times 2 \times \pi \times 30)$$
$$= 400 + 188.495...$$
$$= 588.495...$$
$$\approx 588.5 \text{ m}$$

c $$\text{Cost} = \$124 \times 588.5$$
$$= \$72\,974$$

Question 14

a Area of frame = Area 1 – area 2

Area 1 = $l \times w$
$= 90 \times 70$
$= 6300 \text{ cm}^2$

For Area 2:

$l = 90 - 16 - 16$ $\quad$ $b = 70 - 16 - 16$
$= 58$ $\quad$ $= 38$

Area 2 = $l \times w$
$= 58 \times 38$
$= 2204 \text{ cm}^2$

$\therefore$ Area = $6300 \text{ cm}^2 - 2204 \text{ cm}^2$
$= 4096 \text{ cm}^2$

Convert to m^2: $\div 100 \div 100 = 0.4096 \text{ m}^2$

b 0.4096 m^2 at a cost of \$135 per metre.
$0.4096 \times 135 = \$55.30$

Question 15

a **i** $\text{Radius} = \frac{5}{2} = 2.5 \text{ cm}$

$$\begin{aligned} A_{\text{shaded}} &= A_{\text{square}} - 4 \times A_{\text{semi-circle}} \\ &= 10 \times 10 - 4 \times \frac{1}{2} \times \pi \times 2.5^2 \\ &= 60.730\,09... \\ &\approx 60.7 \text{ cm}^2 \end{aligned}$$

ii $\text{Radius} = \frac{8}{2} = 4 \text{ cm}$

$$\begin{aligned} A_{\text{shaded}} &= A_{\text{square}} - 4 \times A_{\text{semi-circle}} + 4 \times A_{\text{semi-circle}} \\ &= 16 \times 16 - 4 \times \frac{1}{2} \times \pi \times 4^2 + 4 \times \frac{1}{2} \times \pi \times 4^2 \\ &= 256 \text{ cm}^2 \end{aligned}$$

b The semicircles that lie outside of the square in part **ii** fit exactly into the semicircular cut-outs from within the square. So the area required is just the area of the square.

Question 1

a Area 1 $= \frac{1}{2} \times 20 \times 15 = 150$

Area 2 $= \frac{1}{2} \times (15 + 11) \times 18 = 234$

Area 3 $= \frac{1}{2} \times 11 \times 8 = 44$

$\therefore$ Total area = 150 + 234 + 44
= 428 m^2
≈ 430 m^2 correct to 2 sig fig

b Area 1 $= \frac{1}{2} \times 12 \times 14 = 84$

Area 2 $= \frac{1}{2} \times (12 + 5) \times (10 + 16 + 8) = 289$

Area 3 $= \frac{1}{2} \times 5 \times 10 = 25$

Area 4 $= \frac{1}{2} \times 24 \times 18 = 216$

Area 5 $= \frac{1}{2} \times (18 + 12) \times 16 = 240$

Area 6 $= \frac{1}{2} \times 12 \times 18 = 108$

$\therefore$ Total area = 184 + 289 + 25 + 216 + 240 + 108
= 962 m^2
$\approx$ 960 m^2 correct to 2 sig fig

c Area 1 $= \frac{1}{2} \times 60 \times 105 = 3150$

Area 2 $= \frac{1}{2} \times 45 \times 58 = 1305$

Area 3 $= \frac{1}{2} \times 60 \times 58 = 1740$

$\therefore$ Total area = 3150 + 1305 + 1740
= 6195 m^2
≈ 6200 m^2 correct to 2 sig fig

d Area 1 $= \frac{1}{2} \times 20 \times 20 = 200$

Area 2 $= \frac{1}{2} \times 20 \times 30 = 300$

Area 3 $= \frac{1}{2} \times (30 + 40) \times 46 = 1610$

Area 4 $= \frac{1}{2} \times (20 + 25) \times 42 = 945$

Area 5 $= \frac{1}{2} \times 40 \times 20 = 400$

Area 6 $= \frac{1}{2} \times 25 \times 24 = 300$

$\therefore$ Total area = 200 + 300 + 1610 + 945 + 400 + 300
= 3755 m^2
≈ 3800 m^2 correct to 2 sig fig

e Area 1 $= \frac{1}{2} \times 35 \times 14 = 245$

Area 2 $= \frac{1}{2} \times 32 \times 30 = 480$

Area 3 $= \frac{1}{2} \times (35 + 14) \times 30 = 735$

Area 4 $= \frac{1}{2} \times 30 \times 37 = 555$

Area 5 $= \frac{1}{2} \times (14 + 18) \times 16 = 256$

Area 6 $= \frac{1}{2} \times 9 \times 18 = 81$

$\therefore$ Total area = 245 + 480 + 735 + 555 + 256 + 81
= 2352 m^2
≈ 2400 m^2 correct to 2 sig fig

Question 2

a Area 1 $= \frac{1}{2} \times 8 \times 16 = 64$

Area 2 $= \frac{1}{2} \times 16 \times 15 = 120$

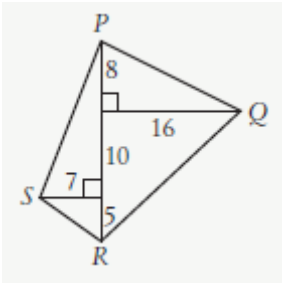
Area 3 $= \frac{1}{2} \times 7 \times 18 = 63$

Area 4 $= \frac{1}{2} \times 7 \times 5 = 17.5$

$\therefore$ Total area = 64 + 120 + 63 + 17.5

$= 264.5 \text{ m}^2$

$\approx 265 \text{ m}^2$



b Area 1 $= \frac{1}{2} \times 18 \times 36 = 324$

Area 2 $= \frac{1}{2} \times (36 + 15) \times 14 = 357$

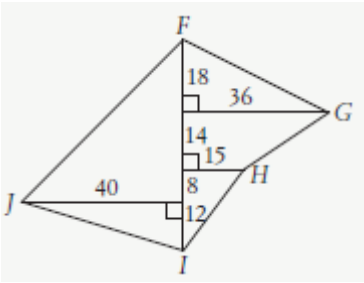
Area 3 $= \frac{1}{2} \times 15 \times 20 = 150$

Area 4 $= \frac{1}{2} \times 40 \times 12 = 240$

Area 5 $= \frac{1}{2} \times 40 \times 40 = 800$

$\therefore$ Total area = 324 + 357 + 150 + 240 + 800

$= 1871 \text{ m}^2$



c Area 1 $= \frac{1}{2} \times 12 \times 28 = 168$

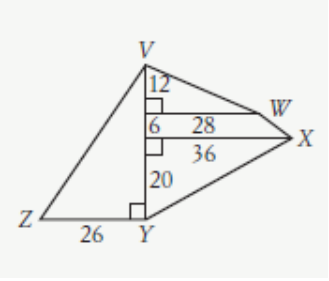
Area 2 $= \frac{1}{2} \times (28 + 36) \times 6 = 192$

Area 3 $= \frac{1}{2} \times 36 \times 20 = 360$

Area 4 $= \frac{1}{2} \times 38 \times 26 = 494$

$\therefore$ Total area = 168 + 192 + 360 + 494

$= 1214 \text{ m}^2$



Question 3

a Find the length of AM using Pythagoras.

To find unknown straight side use the hypotenuse 58 m and side 32 m.

$58^2 = x^2 + 32^2$

$x^2 = 58^2 - 32^2$

$x^2 = 2340$

$x = \sqrt{2340}$

$x = 48.37...$ m

$\therefore$ Area 1 $= \frac{1}{2} \times 32 \times 48.37... = 773.9...$

Area 2 $= \frac{1}{2} \times 16 \times 48.37... = 386.9...$

$\therefore$ Total area $= 773.9... + 386.9...$

$= 1160.8$

$\approx 1161 \text{ m}^2$

b Using half of triangle *AMR*, use Pythagoras to find the hypotenuse *AR*.

The sides are 16 m and $\frac{\sqrt{2340}}{2} \approx 24.186...$ m.

$x^2 = 16^2 + (24.186...)^2$

$x^2 = 841$

$x = \sqrt{841}$

$x = 29$ m

$\therefore$ Perimeter $= 58 + 32 + 2 \times 29$

$= 148 \text{ m}^2$

Question 4

$A = \frac{h}{2}(d_f + d_l)$

Area $= \frac{3}{2}(5 + 6)$

$= 16.5$

$\approx 17 \text{ m}^2$

$\therefore$ B

Question 5

a $A = \frac{h}{2}(d_f + d_l)$

$$\text{Area} = \frac{18}{2}(21+12)$$
$$= 297 \text{ m}^2$$

b $A = \frac{h}{2}(d_f + d_l)$

$$\text{Area} = \frac{30}{2}(16+28)$$
$$= 660 \text{ m}^2$$

Question 6

a $A = \frac{h}{2}(d_f + d_l)$

$$\text{Area 1} = \frac{5}{2}(8+6) = 35$$

$$\text{Area 2} = \frac{5}{2}(6+4.5) = 26.25$$

$$\therefore \text{ Area} = 35 + 26.25$$
$$= 61.25 \text{ m}^2$$
$$\approx 61 \text{ m}^2$$

b $61.25 \div 12 = 5.01...$

$\therefore$ Need 6L of paint

$$\therefore \text{ Cost} = 6 \times \$45$$
$$= \$270$$

Question 7

$A = \frac{h}{2}(d_f + d_l)$

$$\text{Area 1} = \frac{4}{2}(0+7) = 14$$

$$\text{Area 2} = \frac{4}{2}(7+8) = 30$$

$$\text{Area 3} = \frac{4}{2}(8+12) = 40$$

$$\text{Area 4} = \frac{4}{2}(12+10) = 44$$

$$\therefore \text{ Area} = 14 + 30 + 40 + 44$$
$$= 128 \text{ m}^2$$

Question 8

$A = \frac{h}{2}(d_f + d_l)$

$\text{Area } 1 = \frac{6}{2}(2.5 + 4) = 19.5$

$\text{Area } 2 = \frac{6}{2}(4 + 1.8) = 17.4$

$\text{Area } 3 = \frac{6}{2}(1.8 + 7.2) = 27$

$\text{Area } 4 = \frac{6}{2}(7.2 + 7) = 42.6$

$\therefore \text{ Area } = 19.5 + 17.4 + 27 + 42.6$

$= 106.5 \text{ m}^2$

Exercise 5.08 Volume and surface area of a prism

Question 1

a $7\times100^3 = 7\,000\,000\ \text{cm}^2$

b $50\times10^3 = 50\,000\ \text{mm}^2$

c $89\,000\div100^3 = 0.089\ \text{m}^2$

d $0.468\times100^3 = 468\,000\ \text{cm}^2$

e $2400\div10^3 = 2.4\ \text{cm}^2$

f $5\,600\,000\div100^3 = 5.6\ \text{m}^2$

g $9\,100\,000\div10^3 = 9100\ \text{cm}^2$

h $12\times100^3 = 12\,000\,000\ \text{cm}^2$

Question 2

$1000\ \text{cm}^3 = 1000\ \text{mL} = 1\text{L}$

$1\ \text{cm}^3 = 1\ \text{mL}$

$1\ \text{m}^3 = 1\ \text{kL} = 1000\ \text{L}$

1 ML = 1 million litres

a $680\ \text{cm}^3 = 680\,\text{mL}$

b $8500\ \text{cm}^3 = 8500\ \text{mL}$
$= 8500\div1000\ \text{L}$
$= 8.5\ \text{L}$

c $22\ \text{m}^3 = 22\times1000\ \text{L}$
$= 22\,000\ \text{L}$

d $8000\ \text{L} = 8000\div1000\ \text{kL}$
$= 8\ \text{kL}$
$= 8\ \text{m}^3$

e $3.5\ \text{m}^3 = 3.5\times1000\text{L}$
$= 3500\ \text{L}$
$= 3500\times1000\ \text{mL}$
$= 3\,500\,000\ \text{mL}$
$= 3.5\times10^6\ \text{mL}$

f $690\ \text{L} = 690\times1000\ \text{mL}$
$= 690\,000\ \text{mL}$
$= 690\,000\ \text{cm}^3$

g $55\ \text{m}^3 = 55\times1000\ \text{L}$
$= 55\,000\ \text{L}$

h $4300\ \text{m}^3 = 4300\ \text{kL}$

i $9500\ \text{L} = 9500\div1000\ \text{m}^3$
$= 9.5\ \text{m}^3$

j $8.5\times10^4\ \text{cm}^3 = 8.5\times10^4\ \text{mL}$
$= 8.5\times10^4\div1000\ \text{L}$
$= 85\ \text{L}$

k $4.3\times10^{-3}\ \text{kL} = 4.3\times10^{-3}\times1000\ \text{L}$
$= 4.3\ \text{L}$
$= 4.3\times1000\ \text{mL}$
$= 4300\ \text{mL}$
$= 4300\ \text{cm}^3$

l $10^6\ \text{m}^3 = 10^6\ \text{kL}$
$= 10^6\div1000\ \text{ML}$
$= 10^3\ \text{ML}$
$= 1000\ \text{ML}$

Question 3

$V = Ah$
$= (1 \times 0.6) \times 2$
$= 1.2 \text{ m}^3$

$\therefore$ A

Question 4

a Change all measurements to m.

80 cm = 0.8 m and 90 cm = 0.9 m

$\therefore V = Ah$
$= (1.2 \times 0.8) \times 0.9$
$= 0.864 \text{ m}^3$

b $0.864 \text{ m}^3 = 0.864 \times 1000 \text{ L}$
$= 864 \text{ L}$

Question 5

$V = Ah$
$= (2.5 \times 2.5) \times 2.5$
$= 15.625 \text{ m}^3$

$\therefore$ B

Question 6

a $V = (5 \times 5) \times 10$
$= 250 \text{ cm}^3$

b $V = (\frac{1}{2} \times 0.9 \times 1.8) \times 2.1$
$= 1.701 \text{ m}^3$

c $V = (\frac{1}{2} \times (15 + 18) \times 8) \times 2.3$
$= 3036 \text{ cm}^3$

d $V = (0.8 \times 6) \times 4.2$
$= 20.16 \text{ m}^3$

Question 7

$V = (57 \times 81) \times 54$
$= 249\,318 \text{ cm}^3$
$= 249.318 \text{ L}$
$\approx 249 \text{ L}$

$\therefore$ Capacity is 249 L.

Question 8

a $V = 11.5 \times 1.4$
$= 16.1 \text{ m}^3$

b $16.1 \text{ m}^3 = 16.1 \times 1000 \text{ L}$
$= 16\ 100 \text{ L}$

Question 9

a $V = \left[\frac{1}{2} \times (1.4 + 1.8) \times 1\right] \times 2.5$
$= 4 \text{ m}^3$

b For 1 skip = \$16.50 × 4
= \$66

For 4 skips = \$66 × 4
= \$264

Question 10

a $2.031 \times 10^6 \text{ ML} = 2.031 \times 10^6 \times 1\ 000\ 000 \text{ L}$
$= 2.031 \times 10^{12} \text{ L}$

b $2.031 \times 10^{12} \text{ L} = 2.031 \times 10^{12} \div 1000 \text{ m}^3$
$= 2.031 \times 10^9 \text{ m}^3$

Question 11

$\text{SA} = 1 \times 2 + 2 \times (1 \times 0.6) + 2 \times (2 \times 0.6)$
$= 5.6 \text{ m}^2$

$\therefore$ C

Question 12

a $\text{SA} = 2 \times (8 \times 6) + 2 \times (6 \times 5) + 2 \times (8 \times 5)$

$= 236 \text{ m}^2$

b $\text{SA} = 6 \times (7.6^2)$

$= 346.56$

$\approx 347 \text{ m}^2$

c $\text{SA} = 2 \times (5.4^2) + 4 \times (6.8 \times 5.4)$

$= 205.2$

$\approx 205 \text{ m}^2$

d Use Pythagoras to find the hypotenuse of the triangle. It has sides of 3 m and 8 m.

$x^2 = 3^2 + 8^2$

$x^2 = 73$

$x = \sqrt{73}$

$x = 8.54...$

$\text{SA} = 8.54... \times 15 + 3 \times 15 + 8 \times 15 + 2 \times \left(\frac{1}{2} \times 3 \times 8 \right)$

$= 317.1...$

$\approx 317 \text{ m}^2$

e Use Pythagoras to find the height of the triangle. It has a hypotenuse of 10 m and a side of $\frac{12}{2} = 6$ m.

$10^2 = x^2 + 6^2$

$x^2 = 10^2 - 6^2$

$x^2 = 64$

$x = \sqrt{64}$

$x = 8 \text{ m}$

$\text{SA} = 2 \times (10 \times 20) + 12 \times 20 + 2 \times \left(\frac{1}{2} \times 12 \times 8 \right)$

$= 736 \text{ m}^2$

Question 13

a Use Pythagoras to find the height of the triangle. It has a hypotenuse of 10 m and a side of $\frac{12}{2} = 6$ m.

$$\begin{aligned} 50^2 &= x^2 + 30^2 \\ x^2 &= 50^2 - 30^2 \\ x^2 &= 1600 \\ x &= \sqrt{1600} \\ x &= 40 \text{ cm} \end{aligned}$$

Convert all measurements to metres.

$$\begin{aligned} \text{SA} &= 2 \times \left(\frac{1}{2} \times 0.6 \times 0.4 \right) + 2 \times (0.5 \times 1.5) \\ &= 1.74 \\ &\approx 1.7 \text{ m}^2 \end{aligned}$$

b $V = Ah$

$$\begin{aligned} &= \left(\frac{1}{2} \times 0.6 \times 0.4 \right) \times 1.5 \\ &= 0.18 \text{ m}^3 \end{aligned}$$

$$\begin{aligned} \therefore \text{Capacity} &= 0.18 \times 1000 \text{ L} \\ &= 180 \text{ L} \end{aligned}$$

Question 14

a $$\begin{aligned} A &= 0.4 \times (0.94) + 0.32 \times (0.62) + 0.3 \times 0.3 \\ &= 0.6644 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} P &= 0.3 + 0.3 + 0.32 + 0.32 + 0.32 + 0.40 + 0.94 + 1.02 \\ &= 3.92 \text{ m} \end{aligned}$$

$h = 1.2$ m (given)

$$\begin{aligned} \therefore S &= 2A + Ph \\ &= 2 \times 0.6644 + 3.92 \times 1.2 \\ &= 6.0328 \\ &\approx 6 \text{ m}^2 \end{aligned}$$

b $$\begin{aligned} V &= Ah \\ &= 0.6644 \times 1.2 \\ &= 0.79728 \\ &\approx 0.8 \text{ m}^3 \end{aligned}$$

Question 15

a $V = Ah$

$= \left(\frac{1}{2} \times 1.2 \times 1.5\right) \times 2$

$= 1.8 \text{ m}^3$

b Need to find the length of the sides of the tent using Pythagoras.

The sides of the triangle are 1.2 m and $(1.5 \text{ m} \div 2 = 0.75 \text{ m})$.

$x^2 = 1.2^2 + (0.75)^2$

$x^2 = 2.0025$

$x = \sqrt{2.0025}$

$x = 1.415... \text{ m}$

$\text{Area}_{\text{side}} = 1.415... \times 2 = 2.83...$

$\text{Area}_{\text{side}} = 1.415... \times 2 = 2.83...$

$\text{Area}_{\text{floor}} = 1.5 \times 2 = 3$

$\text{Area}_{\text{back}} = \frac{1}{2} \times 1.5 \times 1.2 = 0.9$

$\text{Area}_{\text{front}} = \frac{1}{2} \times 1.5 \times 1.2 = 0.9$

$\therefore \text{Total area} = 2.83... + 2.83... + 3 + 0.9 + 0.9$

$\approx 10.5 \text{ m}^2$

Question 16

a $V = Ah$

$= \left(\frac{1}{2} \times 3 \times 14\right) \times 4$

$= 84 \text{ cm}^3$

b Use Pythagoras to find the hypotenuse of the triangle. It has sides of 3 cm and 814 cm.

$x^2 = 3^2 + 14^2$

$x^2 = 205$

$x = \sqrt{205}$

$x = 14.317...$

Surface area of one wedge:

$\text{SA} = 3 \times 4 + 4 \times 14.317... + 4 \times 14 + 2 \times \left(\frac{1}{2} \times 14 \times 3\right)$

$= 167.27... \text{ cm}^2$

$= 167.27... \div 100^2 \text{ m}^2$

$= 0.016727... \text{ m}^2$

5 pots can do 5 m^2

$\therefore$ Number of wedges $= 5 \div 0.016727...$

$= 298.9...$

$\therefore$ Kobi can varnish 298 wedges with five pots.

Exercise 5.09 Volume and surface area of a cylinder

Question 1

a $V = \pi r^2 h$
$= \pi \times 7^2 \times 12$
$= 1847.256...$
$\approx 1800 \text{ m}^3$ (correct to 2 sig fig)

b Change 40 cm to m, i.e., 0.4 m
$V = \pi r^2 h$
$= \pi \times 0.4^2 \times 2$
$= 1.005...$
$\approx 1.0 \text{ m}^3$ (correct to 2 sig fig)

c $\text{Radius} = \frac{12}{2} = 6$ cm
$V = \pi r^2 h$
$= \pi \times 6^2 \times 24$
$= 2714.336...$
$\approx 2700 \text{ m}^3$ (correct to 2 sig fig)

d $\text{Radius} = \frac{1.3}{2} = 0.65$ m
Change 20 cm to m, i.e, 0.2 m.
$V = \pi r^2 h$
$= \pi \times 0.65^2 \times 0.2$
$= 0.2654...$
$\approx 0.27 \text{ m}^3$ (correct to 2 sig fig)

e $V = \pi r^2 h$
$= \pi \times 2.4^2 \times 2.4$
$= 43.42...$
$\approx 43 \text{ m}^3$ (correct to 2 sig fig)

f $\text{Radius} = \frac{1.7}{2} = 0.85$ m
$V = \pi r^2 h$
$= \pi \times 0.85^2 \times 1.3$
$= 2.950...$
$\approx 3.0 \text{ m}^3$ (correct to 2 sig fig)

Question 2

a $SA = 2\pi r^2 + 2\pi rh$
$= 2 \times \pi \times 7^2 + 2 \times \pi \times 7 \times 12$
$= 835.66...$
$\approx 840 \text{ m}^2$ (correct to 2 sig fig)

b Change 40 cm to m, i.e., 0.4 m
$SA = 2\pi r^2 + 2\pi rh$
$= 2 \times \pi \times 0.4^2 + 2 \times \pi \times 0.4 \times 2$
$= 6.031...$
$\approx 6.0 \text{ m}^2$

c $\text{Radius} = \frac{12}{2} = 6$ cm
$SA = 2\pi r^2 + 2\pi rh$
$= 2 \times \pi \times 6^2 + 2 \times \pi \times 6 \times 24$
$= 1130.97...$
$\approx 1100 \text{ cm}^2$

d $\text{Radius} = \frac{1.3}{2} = 0.65$ m
Change 20 cm to m, i.e., 0.2 m
$SA = 2\pi r^2 + 2\pi rh$
$= 2 \times \pi \times 0.65^2 + 2 \times \pi \times 0.65 \times 0.2$
$= 3.471...$
$\approx 3.5 \text{ m}^2$

e $SA = 2\pi r^2 + 2\pi rh$
$= 2 \times \pi \times 2.4^2 + 2 \times \pi \times 2.4 \times 2.4$
$= 72.382...$
$\approx 72 \text{ m}^2$

f $\text{Radius} = \frac{1.7}{2} = 0.85$ m
$SA = 2\pi r^2 + 2\pi rh$
$= 2 \times \pi \times 0.85^2 + 2 \times \pi \times 0.85 \times 1.3$
$= 11.482...$
$\approx 11 \text{ m}^2$

Question 3

a $\text{Radius} = \frac{8}{2} = 4 \text{ cm}$

$V = \pi r^2 h$

$= \pi \times 4^2 \times 6.5$

$= 326.725...$

$\approx 327 \text{ cm}^3$

b $\text{Engine capacity} = 6 \times 326.725...$

$= 1960.... \text{ cm}^3$

$= 1960.... \text{ mL}$

$= 1960.... \div 1000 \text{ L}$

$= 1.96... \text{ L}$

$\approx 2 \text{ L}$

Question 4

a $V = \pi r^2 h$

$= \pi \times 15^2 \times 10$

$= 7068.58...$

$\approx 7100 \text{ cm}^3$

b $\text{Radius} = \frac{1.2}{2} = 0.6 \text{ m}$

$V = \frac{1}{2} \pi r^2 h$

$= \frac{1}{2} \times \pi \times 0.6^2 \times 1.5$

$= 0.8482...$

$\approx 0.85 \text{ m}^3$

c $\text{Radius} = \frac{4}{2} = 2 \text{ m}$

$V = \frac{1}{2} \pi r^2 h$

$= \frac{1}{2} \times \pi \times 2^2 \times 5$

$= 31.415...$

$\approx 31 \text{ m}^3$

d $R = \frac{11}{2} = 5.5 \text{ cm}$

$r = \frac{8}{2} = 4 \text{ cm}$

Length = 2 m = 200 cm

$V = \pi R^2 h - \pi r^2 h$

$= \pi \times 5.5^2 \times 200 - \pi \times 4^2 \times 200$

$= 19\ 006.63... - 10\ 053.09...$

$= 8953.53...$

$\approx 9000 \text{ cm}^3$

Question 5

$R \text{ (bottom layer)} = \dfrac{50}{2} = 25$

$r \text{ (top layer)} = \dfrac{30}{2} = 15$

Top layer: $SA = \pi r^2 + 2\pi rh$

$= \pi \times 15^2 + 2 \times \pi \times 15 \times 12$

$\approx 1837.83 \text{ cm}^2$

Bottom layer: $SA = \pi \text{R}^2 - \pi r^2 + 2\pi Rh$

$= \pi \times 25^2 - \pi \times 15^2 + 2 \times \pi \times 25 \times 15$

$= 3612.83... \text{ cm}^2$

$\therefore \text{Area of icing} = 1837.83... + 3612.83...$

$= 5450...$

$\approx 5451 \text{ cm}^2$

Question 6

a $r = \dfrac{2}{2} = 1 \text{ m}$

$V = \dfrac{1}{2}\pi r^2 h$

$= \dfrac{1}{2} \times \pi \times 1^2 \times 2$

$= 3.141...$

$\approx 3.1 \text{ m}^3$

b $\text{Surface area} = \dfrac{1}{2}\left(2\pi rh + 2\pi r^2\right) + 2 \times 2$

$= \dfrac{1}{2}\left(2 \times \pi \times 1 \times 2 + 2 \times \pi \times 1^2\right) + 2 \times 2$

$= 13.42...$

$\approx 13.4 \text{ m}^2$

Question 7

a $\text{r}_{\text{hole}} = \dfrac{4 + 0.5 + 0.5}{2} = 2.5 \text{ m}$

$V_{\text{hole}} = \pi r^2 h$

$= \pi \times 2.5^2 \times 2$

$= 39.269...$

$\approx 39 \text{ m}^3$

b $\text{r}_{\text{tank}} = \dfrac{4}{2} = 2 \text{ m}$

$V_{\text{tank}} = \pi r^2 h$

$= \pi \times 2^2 \times 2$

$= 25.1327... \text{ m}^3$

$= 25.1327... \times 1000 \text{ L}$

$= 25\,132.7... \text{ L}$

$\approx 25\,133 \text{ L}$

Question 8

$$V_{\text{original}} = \pi r^2 h$$
$$= \pi \times 5^2 \times h$$
$$= 25\pi h$$

$$V_{\text{new}} = \pi r^2 h$$
$$= \pi \times 10^2 \times h$$
$$= 100\pi h$$

$$\frac{V_{\text{new}}}{V_{\text{original}}} = \frac{100\pi h}{25\pi h}$$
$$= 4$$

$\therefore$ The volume is four times larger.

$\therefore$ **C**.

Exercise 5.10 Volume and surface area of a sphere

Question 1

a $V = \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 42^3$

$\approx 310\,000 \text{ km}^3$

b $V = \frac{1}{2} \times \frac{4}{3}\pi r^3$

$= \frac{1}{2} \times \frac{4}{3} \times \pi \times 47.5^3$

$\approx 224\,000 \text{ m}^3$

c $V = \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 600^3$

$\approx 9.05 \times 10^8 \text{ m}^3$

d $V = \frac{1}{2} \times \frac{4}{3}\pi r^3$

$= \frac{1}{2} \times \frac{4}{3} \times \pi \times 78^3$

$\approx 993\,899 \text{ mm}^3$

$= 993\,899 \div 1000 \text{ cm}^3$

$\approx 994 \text{ cm}^3$

e $V = \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 83.5^3$

$\approx 2\,438\,641 \text{ cm}^3$

$= 2\,438\,641 \div 1\,000\,000 \text{ m}^3$

$\approx 2.44 \text{ m}^3$

f $V = \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 14^3$

$\approx 11\,500 \text{ m}^3$

Question 2

a **i** Radius of Earth = 12 683 ÷ 2 = 6341.5 km

Surface area of Earth $= 4\pi r^2$

$= 4 \times \pi \times 6341.5^2$

$= 505\,351\,847.3... \text{ km}^2$

If $\frac{4}{5}$ is covered by water, then $\frac{1}{5}$ is covered by land.

So, Area covered by land $= \frac{1}{5} \times 505\,351\,847.3...$

$= 101\,070\,369.5...$

$\approx 1.011 \times 10^8 \text{ km}^2$

ii $V = \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 6341.5^3$

$\approx 1.068 \times 10^{12} \text{ km}^3$

b Mass per cubic kilometre $= \frac{5.974 \times 10^{21} \text{ t}}{1.068 \times 10^{12} \text{ km}^3}$

$\approx 5.592 \times 10^9 \text{ t/km}^3$

Question 3

a
$$V = \frac{1}{2} \times \frac{4}{3}\pi r^3$$
$$= \frac{1}{2} \times \frac{4}{3} \times \pi \times 0.5^3$$
$$\approx 0.2618 \text{ m}^3$$
$$= 0.2618 \times 1000 \text{ L}$$
$$\approx 262 \text{ L}$$

b
$$\text{Surface area} = \frac{1}{2} \times 4\pi r^2$$
$$= \frac{1}{2} \times 4 \times \pi \times 0.5^2$$
$$\approx 1.57 \text{ m}^2$$

Question 4

a
$$V_{\text{bauble}} = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} \times \pi \times 3^3$$
$$\approx 113 \text{ cm}^3$$

b
$$V_{\text{box}} = 6 \times 6 \times 6$$
$$= 216 \text{ cm}^3$$

$$V_{\text{air}} = V_{\text{box}} - V_{\text{bauble}}$$
$$= 216 - 113$$
$$= 103 \text{ cm}^3$$

c
$$\text{Surface area of one bauble} = 4\pi r^2$$
$$= 4 \times \pi \times 3^2$$
$$= 113.0973 \ldots \text{ cm}^2$$

$$\text{S.A of 100 baubles} = 113.0973 \ldots \times 100$$
$$= 11\,309.73 \ldots \text{ cm}^2$$
$$= 11\,309.73 \ldots \div 10\,000 \text{ m}^2$$
$$= 1.130973 \ldots \text{ m}^2$$

$$\therefore \text{Cost to cover 100 baubles} = 1.130973 \ldots \times \$15$$
$$= \$16.964 \ldots$$
$$\approx \$16.96$$

Question 5

$$V = \frac{4}{3}\pi r^3$$

$$400 = \frac{4}{3}\pi r^3$$
$$400 \times 3 = 4\pi r^3$$
$$1200 = 4\pi r^3$$
$$\frac{1200}{4\pi} = r^3$$
$$r^3 = 95.4929 \ldots$$
$$r = \sqrt[3]{95.4929 \ldots}$$
$$r = 4.5707 \ldots$$
$$r \approx 4.6$$

The correct answer is **A**.

Question 6

a $V = \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 28^3$

$\approx 92\,000 \text{ cm}^3$

b Surface area $= 4\pi r^2$

$= 4 \times \pi \times 28^2$

$\approx 9852 \text{ cm}^2$

Each tile covers 1 cm^2, so it takes 9852 tiles to cover the surface of the ball.

Question 7

a Find the surface area of a sphere rather than that of two half spheres.

Surface area $= 4\pi r^2$

$= 4 \times \pi \times 0.48^2$

$\approx 2.9 \text{ m}^2$

b $A = 2 \times \pi r^2$

$= 2 \times \pi \times 0.48^2$

$\approx 1.4 \text{ m}^2$

c Find the volume of one sphere rather than two half spheres.

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 0.48^3$

$\approx 0.46 \text{ m}^3$

Question 1

a $r = \frac{75}{2} = 37.5 \text{ mm}$ $\quad h = 3 \times 75 = 225 \text{ mm}$

$$\begin{aligned} V_{\text{can}} &= \pi r^2 h \\ &= \pi \times 37.5^2 \times 225 \\ &= 994\ 019.55... \\ &\approx 994\,020 \text{ mm}^3 \end{aligned}$$

b $V_{\text{balls}} = 3 \times \frac{4\pi}{3}\left(\frac{75}{2}\right)^3 \approx 662\ 680 \text{ mm}^3$

$$\frac{V_{\text{balls}}}{V_{\text{can}}} = \frac{662\ 680}{994\,020} = \frac{2}{3}$$

Roughly 67% of the can's volume is taken up by tennis balls.

c The square-based prism would have a base with side length 75 mm (which is the diameter of a ball) and a height of 225 mm (which is the sum of the diameters of three balls).

$$\begin{aligned} V &= Ah \\ &= 75^2 \times 225 \\ &= 1\,265\,625 \text{ mm}^3 \end{aligned}$$

d The volume of the cylinder containing the balls is less than that of a square prism containing the balls. The cylinder is the most efficient method of packaging because it contains the least amount of air.

Question 2

a $V = Ah$

$= (110 \times 120) \times 130$

$= 1\,716\,000 \text{ mm}^3$

b $r = \frac{20}{2} = 10 \text{ mm}$

$V_{\text{hole}} = \pi r^2 h$

$= \pi \times 10^2 \times 130$

$= 40\,840.70... \text{ mm}^3$

$V_{\text{after drilling}} = 1\,716\,000 - 3 \times 40\,840.70...$

$= 1\,593\,477.8... \text{mm}^3$

$= 1\,593\,477.8... \div 10^3 \text{ cm}^3$

$= 1593.4... \text{cm}^3$

$\approx 1593 \text{cm}^3$

c $\%\,\text{removed} = \frac{3 \times 40\,840.70...}{1\,716\,000} \times 100\%$

$= 7.13...\%$

$\approx 7\%$

d $\text{SA} = 2 \times A_{\text{front}} + 2 \times A_{\text{side}} + 2 \times A_{\text{top}}$

$= 2 \times 110 \times 120 + 2 \times 130 \times 120 + 2 \times 110 \times 130$

$= 86\,200 \text{ mm}^2$

Question 3

a $r = \frac{38}{2} = 19 \text{ mm}$

$$V_{\text{whole}} = \pi r^2 h$$
$$= \pi \times 19^2 \times 8.5$$
$$= 9639.9... \text{ cm}^3$$

$$V_{\text{remaining}} = \frac{7}{8} \times 9639.9...$$
$$= 8434.9...$$
$$\approx 8400 \text{ cm}^3$$

b The top and bottom of the slice are both sectors with $\frac{7}{8}^{\text{ths}}$ of the circle.

(NOTE: Could use angle of $\frac{360}{8} = 45^\circ$ but still is equal to eighths.)

The sides of the slice are both rectangles with length 19 cm and width 8.5 cm.

$$\therefore SA = 2 \times A_{\text{sector}} + A_{\text{curved section}} + 2 \times A_{\text{rectangle sides}}$$
$$= 2 \times \frac{7}{8} \times \pi r^2 + \frac{7}{8} \times 2\pi rh + 2 \times rh$$
$$= 2 \times \frac{7}{8} \times \pi \times 19^2 + \frac{7}{8} \times 2 \times \pi \times 19 \times 8.5 + 2 \times 19 \times 8.5$$
$$= 1984.7... + 887.8... + 323$$
$$= 3195.59...$$
$$\approx 3200 \text{ cm}^2$$

c $V_{\text{one sector}} = \frac{7}{8} \times V_{\text{whole}}$

$$= \frac{1}{8} \times 9639.9...$$
$$= 1204.99...$$
$$\approx 1200 \text{ cm}^3$$

d $A = 2 \times A_{\text{sector}} + A_{\text{curved section}}$

$$= 2 \times \frac{1}{8} \times \pi r^2 + \frac{1}{8} \times 2\pi rh$$
$$= 2 \times \frac{1}{8} \times \pi \times 19^2 + \frac{1}{8} \times 2 \times \pi \times 19 \times 8.5$$
$$= 410.3...$$
$$\approx 410 \text{ cm}^3$$

Question 4

a Pool A:

$$\begin{aligned} A &= \pi r^2 + lw \\ &= \pi \times 1.5^2 + 3 \times 7 \\ &= 28.068...\ \text{m}^2 \\ V &= Ah \\ &= 28.068... \times 1.5 \\ &= 42.102...\ \text{m}^3 \\ &= 42.102... \times 1000\ \text{L} \\ &= 42\ 102.8...\ \text{L} \\ &\approx 42\ 103\ \text{L} \end{aligned}$$

Pool B:

$$\begin{aligned} A &= lw + \frac{1}{2}(a+b)h \\ &= 6 \times 1.5 + \frac{1}{2} \times (1.5+2) \times 4 \\ &= 16\ \text{m}^2 \\ V &= Ah \\ &= 16 \times 3 \\ &= 48\ \text{m}^3 \\ &= 48 \times 1000\ \text{L} \\ &= 48\ 000\ \text{L} \end{aligned}$$

b $\text{Difference} = 48\ 000 - 42\ 103$
$= 5897\ \text{L}$

$\therefore$ Pool B by 5897 litres.

c From part **a**: $A_{\text{front}} = A_{\text{back}} = 16\ \text{m}^2$

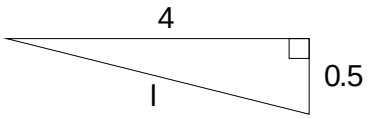
$$\begin{aligned} A_{\text{left side}} &= 1.5 \times 3 \\ &= 4.5\ \text{m}^2 \end{aligned}$$

$$\begin{aligned} A_{\text{right side}} &= 2 \times 3 \\ &= 6\ \text{m}^2 \end{aligned}$$

$$\begin{aligned} A_{\text{flat bottom}} &= 6 \times 3 \\ &= 18\ \text{m}^2 \end{aligned}$$

$$A_{\text{sloped bottom}} = l \times 3$$

Find l using Pythagoras' theorem:



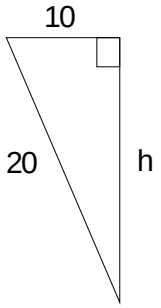
$$\begin{aligned} l &= \sqrt{4^2 + 0.5^2} \\ &= 4.03...\ \text{m} \end{aligned}$$

$$\begin{aligned} A_{\text{sloped bottom}} &= 4.03... \times 3 \\ &= 12..09...\ \text{m}^2 \end{aligned}$$

$$\begin{aligned} \text{Tiled area} &= 16 + 16 + 4.5 + 6 + 18 + 12.09... \\ &= 72.59... \\ &\approx 73\ \text{m}^2 \end{aligned}$$

Question 5

a Find the height of the triangular cut-out using Pythagoras' theorem.



$$h = \sqrt{20^2 - 10^2}$$
$$\approx 17.32... \text{ cm}$$

$$A_{\text{front face}} = A_{\text{rectangle}} - A_{\text{triangle}}$$
$$= 40 \times 68 - \frac{1}{2} \times 20 \times 17.32...$$
$$= 2546.79... \text{ cm}^2$$

$$V = Ah$$
$$= 2546.79... \times 120$$
$$= 305\,615.39... \text{ cm}^3$$
$$= 305\,615.39 \div 100^3 \text{ m}^3$$
$$= 0.3056... \text{ m}^3$$
$$\approx 0.31 \text{ m}^3$$

b $A_{\text{front}} = 2546.79... \text{ cm}^2$

$$A_{\text{bottom}} = 68 \times 120$$
$$= 8160 \text{ cm}^2$$

$$A_{\text{side}} = 40 \times 120$$
$$= 4800 \text{ cm}^2$$

$$A_{\text{top white}} = 24 \times 120$$
$$= 2880 \text{ cm}^2$$

$$A_{\text{top green}} = 20 \times 120$$
$$= 2400 \text{ cm}^2$$

$$\text{SA} = 2 \times A_{\text{front}} + A_{\text{bottom}} + 2 \times A_{\text{side}} + 2 \times A_{\text{top white}} + 2 \times A_{\text{top green}}$$
$$= 2 \times 2546.79... + 8160 + 2 \times 4800 + 2 \times 2880 + 2 \times 2400$$
$$= 33\,413.5...$$
$$\approx 33\,414 \text{ cm}^2$$

Question 6

Area of each triangle $= \frac{1}{2} \times 16 \times 8.4$

$= 67.2 \text{ m}^2$

Area of each side rectangle $= 16 \times 3$

$= 48 \text{ m}^2$

SA of marquee = 4 triangles + 4 rectangles

$= 4 \times 67.2 + 4 \times 48$

$= 460.8 \text{ m}^2$

Sample HSC problem

a 20.7 has 3 significant figures.

b It is written to 1 decimal place so it is correct to ± 0.05 m.
The limits of accuracy are: $20.7 \text{ m} \pm 0.05 = 20.65 \text{ m}$ to 20.75 m.

c $$\begin{aligned} \%\text{ error} &= \frac{0.05}{20.7} \times 100\% \\ &= 0.2415...\% \\ &\approx 0.242\% \end{aligned}$$

Test yourself 5

Question 1

a $28.5 \text{ km} = 28.5 \times 1000 \text{ m}$
$= 28\,500 \text{ m}$

b $6.4 \text{ t} = 6.4 \times 1000 \text{ kg}$
$= 6400 \text{ kg}$

c $340 \text{ mL} = 340 \div 1000 \text{ L}$
$= 0.34 \text{ L}$

Question 2

$43 \text{ ML} = 43 \times 1000 \text{ kL}$
$= 43\,000 \text{ L}$

Question 3

a Absolute error: ± 0.005 m

b Limits of accuracy: $3.66 \pm 0.005 = 3.655$ to 3.665 m

c Percentage error $= \frac{0.005}{3.66} \times 100\% = 0.14\%$

Question 4

To two significant figures:

a 38.915 becomes 39

b 1036 becomes 1000

c 0.00872 becomes 0.0087

d 6 587 200 becomes 6 600 00

Question 5

$\$8\,350\,000\,000 = \$8.35 \times 10^9$

Question 6

$4.6 \times 10^{-6} = 0.000\,004\,6$ mm

Question 7

a Perimeter = 9 + 8 + 9 + 2 + 3 + 4 + 3 + 2
= 40 m

b Need to find the unknown side using Pythagoras.
The sides of the triangle are 52 cm and (119 cm – 80 cm = 39 cm).

$x^2 = 52^2 + 39^2$
$x^2 = 4225$
$x = \sqrt{4225}$
$x = 65$ cm

$\therefore$ Perimeter = 80 + 52 + 119 + 65
= 316 cm

c Perimeter = $\frac{2 \times \pi \times 7}{4} + 7 + 22 + 15$
$\approx$ 55.00 cm

Question 8

a $8400 \text{ mm}^2 = 8400 \div 10^2 \text{ cm}^2$
$= 84 \text{ cm}^2$

b $5.6 \text{ ha} = 5.6 \times 10\,000 \text{ m}^2$
$= 56\,000 \text{ m}^2$

Question 9

a Area = (4 × 2) + (2 × 6) + (3 × 8)
= 8 + 12 + 24
= 44 m^2

b Area = $\frac{1}{2}(32 + 48) \times 26$
= 1040 cm^2

c Area = $\pi \times 9.5^2 - \pi \times 4.2^2$
= 228.11… cm^2
$\approx$ 230 cm^2

Question 10

a $P = r + r + \frac{\theta}{360} \times 2\pi r$

$= 46 + 46 + \frac{35}{360} \times 2 \times \pi \times 46$

$= 92 + 28.09...$

$= 120.09...$

$\approx 120.1 \text{ cm}$

b $A = \frac{\theta}{360} \times \pi r^2$

$= \frac{35}{360} \times \pi \times 46^2$

$= 646.29...$

$\approx 646.3 \text{ cm}^2$

Question 11

a Area 1 $= \frac{1}{2} \times (16 + 10) \times 20 = 260$

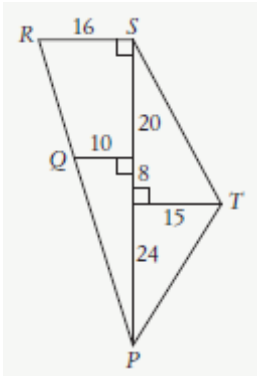
Area 2 $= \frac{1}{2} \times 10 \times 32 = 160$

Area 3 $= \frac{1}{2} \times 28 \times 15 = 210$

Area 4 $= \frac{1}{2} \times 15 \times 24 = 180$

Total area = 260 + 160 + 210 + 180

= 810 m^2



b $810 \text{ m}^2 = 810 \div 10\,000 \text{ ha}$

$= 0.081 \text{ ha}$

Question 12

a $20.7 \text{ cm}^3 = 20.7 \times 10^3 \text{ mm}^3$

$= 20\,700 \text{ mm}^3$

b $1\,650\,000 \text{ cm}^3 = 1\,650\,000 \div 100^3 \text{ m}^3$

$= 1.65 \text{ m}^3$

Question 13

a **i** $V = 4^2 \times \pi \times 15$

$= 753.982...$

$\approx 750 \text{ cm}^3$ (correct to 2 sig fig)

ii $\text{Area}_{\text{side}} = 8 \times \pi \times 15 = 376.99 \text{ cm}^2$

$\text{Area}_{\text{base}} = 4^2 \times \pi = 50.265 \text{ cm}^2$

$\therefore \text{Surface area} = 2 \times 50.265 + 376.99$

$= 477.52... \text{ cm}^2$

$\approx 480 \text{ cm}^2$ (correct to 2 sig fig)

b **i** Convert all measurements to metres.

$V = (\frac{1}{2} \times 0.45 \times 0.48) \times 2$

$= 0.216 \text{ m}^3$

$\approx 0.22 \text{ m}^3$ (correct to 2 sig fig)

ii $\text{Area}_{\text{front}} = \frac{1}{2} \times 0.45 \times 0.48 = 0.108 \text{ m}^2$

Need to find the unknown side using Pythagoras.

The sides of the triangle are 45 cm and $\frac{48}{2} = 24$ cm.

$x^2 = 45^2 + 24^2$

$x^2 = 2601$

$x = \sqrt{2601}$

$x = 51 \text{ cm}$

$= 0.51 \text{ m}$

$\text{Area}_{\text{side}} = 0.51 \times 2 = 1.02 \text{ m}^2$

$\text{Area}_{\text{top}} = 0.48 \times 2 = 0.96 \text{ m}^2$

$\therefore \text{Surface area} = 2 \times 0.108 + 2 \times 1.02 + 0.96$

$= 3.216 \text{ m}^2$

$\approx 3.2 \text{ m}^2$ (correct to 2 sig fig)

Question 14

a $894 \text{ cm}^3 = 894 \text{ mL}$

b $6.5 \text{ m}^3 = 6.5 \times 1000 \text{ L}$

$= 6500 \text{ L}$

Question 15

a $V = \pi \times 1.2^2 \times 1.8$

$= 8.143 \text{ m}^3$

$\approx 8.1 \text{ m}^3$ (correct to 2 sig fig)

b $8.1 \text{ m}^3 = 8.1 \times 1000 \text{ L}$

$= 8100 \text{ L}$

Question 16

a $A \approx \frac{h}{2}\left(d_f + d_l\right)$

$= \frac{22}{2}(6+4)$

$= 110 \text{ m}^2$

b $A \approx \frac{h}{2}\left(d_f + d_l\right)$

$A \approx A_{\text{left trapezium}} + A_{\text{right trapezium}}$

$= \frac{11}{2}(6+12) + \frac{11}{2}(12+4)$

$= 99 + 88$

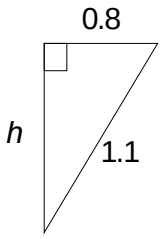
$= 187 \text{ m}^2$

c The 187 m^2 is more accurate, because the more applications (trapeziums) we use, the closer the answer is to the exact area of the field.

Question 17

a Find the height of the trapezium.

$$\text{Side} = \frac{2.9 - 1.3}{2} = 0.8$$



$$h = \sqrt{1.1^2 - 0.8^2}$$
$$= \sqrt{0.57}$$
$$\approx 0.75... \text{ m}$$

Change 90 cm to 0.9 m.

$$A_{\text{top face}} = \frac{1}{2}(a+b)h$$
$$= \frac{1}{2} \times (2.9 + 1.3) \times 0.75...$$
$$= 1.585... \text{ m}^2$$

$$A_{\text{front}} = 1.3 \times 0.9$$
$$= 1.17 \text{ cm}^2$$

$$A_{\text{side}} = 1.1 \times 0.9$$
$$= 0.99 \text{ m}^2$$

$$A_{\text{back}} = 2.9 \times 0.9$$
$$= 2.61 \text{ m}^2$$

$$\text{SA} = 2 \times A_{\text{top}} + A_{\text{front}} + 2 \times A_{\text{side}} + A_{\text{back}}$$
$$= 2 \times 1.585... + 1.17 + 2 \times 0.99 + 2.61$$
$$= 8.93...$$
$$\approx 8.9 \text{ m}^2$$

b $V = Ah$
$$= 1.585... \times 0.9$$
$$= 1.42...$$
$$\approx 1.4 \text{ m}^3$$

Question 18

$r = \frac{2.8}{2} = 1.4 \text{ m}$

$V = \pi r^2 h$
$= \pi \times 1.4^2 \times 7$
$= 43.1026... \text{ m}^3$
$= 43.1026... \times 1000 \text{ L}$
$= 43102.65... \text{ L}$

$\therefore$ Number of bottles $= 43102.65... \text{ L} \div 750 \text{ mL}$

(But need to be in same units, so change 750 mL to 0.75 L.)

$\therefore$ Number of bottles $= 43102.65... \text{ L} \div 0.75 \text{ L}$
$= 57\,470.2...$
$\approx 57\,470$

Question 19

a $V_{\text{ice}} = V_{\text{outer bowl}} - V_{\text{inner bowl}}$
$= \frac{1}{2} \times \frac{4}{3} \times \pi \times 11^3 - \frac{1}{2} \times \frac{4}{3} \times \pi \times 8^3$
$= 2787.639... - 1072.33...$
$= 1715.309...$
$\approx 1715.31 \text{ cm}^3$

b $V_{\text{drink}} = V_{\text{inner bowl}}$
$= \frac{1}{2} \times \frac{4}{3} \times \pi \times 8^3$
$= 1072.33... \text{ cm}^3$
$= 1072.33... \text{ mL}$
$= 1.07233... \text{ L}$
$\approx 1.07 \text{ L}$

Question 20

a Volume rectangular prism $= (4.8 \times 6.5) \times 5.2$

$= 162.24 \text{ m}^3$

Volume of triangular prism $= (\frac{1}{2} \times 4.8 \times 2.7) \times 6.5$

$= 42.12 \text{ m}^3$

$\therefore$ Volume of barn $= 162.24 + 42.12$

$= 204.36 \text{ m}^3$

$\approx 204 \text{ m}^3$

b Area of walls

$\text{Area}_{\text{side}} = 6.5 \times 5.2$

$= 33.8$

$\text{Area}_{\text{front}} = 4.8 \times 5.2$

$= 24.96$

$\therefore \text{Area}_{\text{walls}} = 33.8 + 24.96 + 33.8 + 24.96$

$= 117.52 \text{ m}^2$

Area of roof

Need to find the unknown slant side using Pythagoras.

The sides of the triangle are 2.7 m and $\frac{4.8}{2} = 2.4$ m.

$x^2 = 2.7^2 + 2.4^2$

$x^2 = 13.05$

$x = \sqrt{13.05}$

$x = 3.6124...$ m

$\text{Area}_{\triangle \text{ front}} = \frac{1}{2} \times 2.7 \times 4.8$

$= 6.48$

$\text{Area}_{\text{side}} = 6.5 \times 3.6124...$

$= 23.48...$

$\therefore \text{Area}_{\text{walls}} = 6.48 + 23.48 + 6.48 + 23.48$

$= 59.92 \text{ m}^2$

$\therefore$ Total Area $= 59.92 + 117.52$

$= 177.44 \text{ m}^2$

Two coats $= 177.44 \times 2$

$= 354.88 \text{ m}^2$

Number of litres needed $= 354.88 \div 14$

$= 25.34...$

≈ 26 L

Question 21

a Shape of front is a semi-circle with radius $\frac{10}{2} = 5$ cm and a rectangle that is 10 cm by $18 - 5 = 13$ cm.

$$\begin{aligned}\text{Area}_{\text{face}} &= \frac{1}{2} \times \pi \times 5^2 + 10 \times 13 \\ &= 39.269... + 130 \\ &= 169.269...\end{aligned}$$

$$\begin{aligned}\therefore V &= Ah \\ &= 169.269... \times 28 \\ &= 4739.55... \\ &\approx 4700 \text{ cm}^3 \text{ (correct to 2 sig fig)}\end{aligned}$$

b Length of crust

$$\begin{aligned}&= 13 + 10 + 13 + \text{circumference of semi-circle} \\ &= 36 + \frac{1}{2} \times 2 \times \pi \times 5 \\ &= 36 + 15.707... \\ &= 51.707... \\ &\approx 52 \text{ cm (correct to 2 sig fig)}\end{aligned}$$

