ALGEBRA

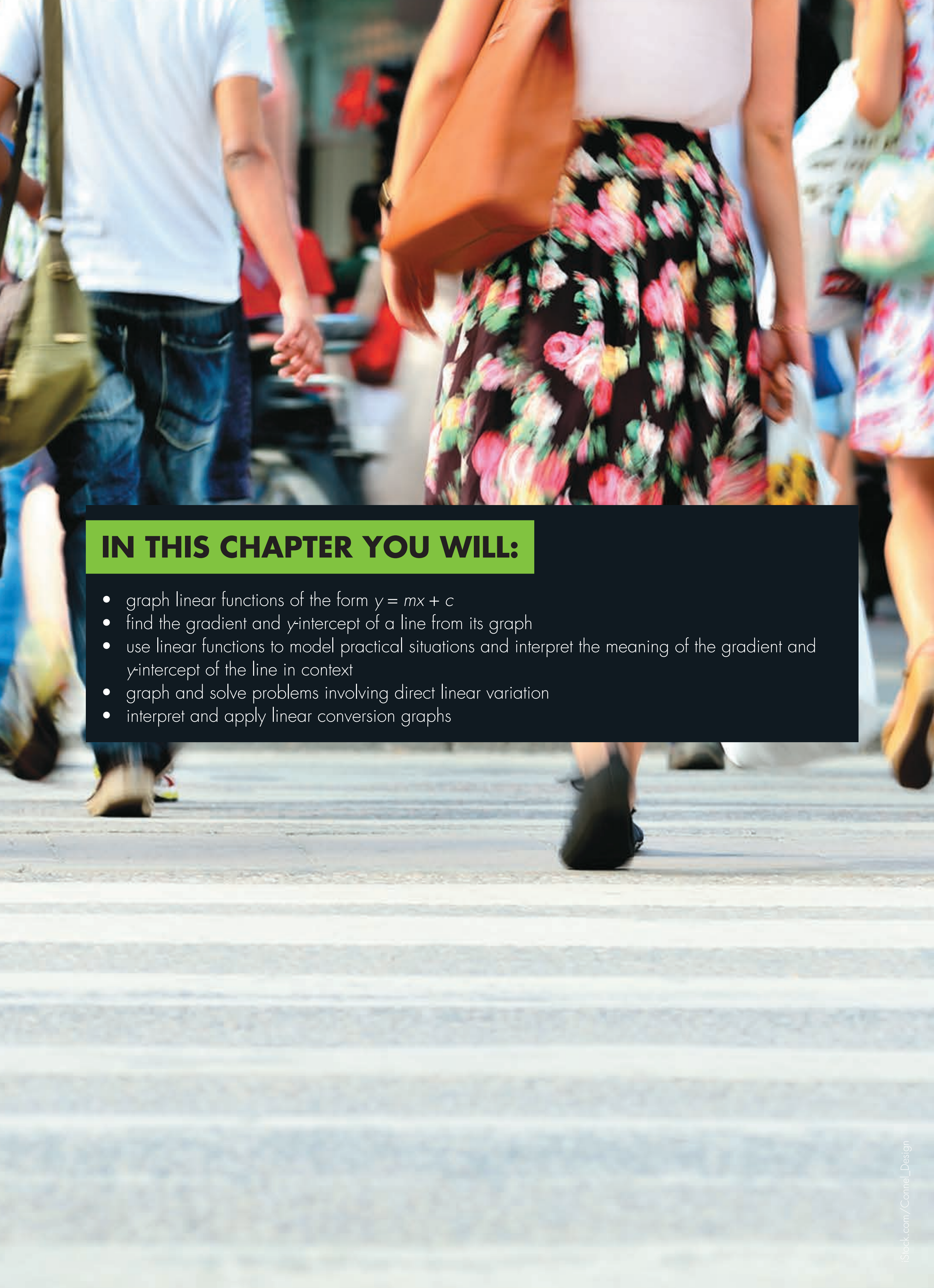
7. LINEAR FUNCTIONS

A criminologist discovered that, in a big city, if the number of police on street patrol was increased, the number of crimes committed decreased. She noticed the following pattern:

Number of police	50	150	200	250	300
Number of crimes (per month)	3100	2800	2650	2500	2350

Is it possible for the criminologist to find an algebraic formula to fit this pattern? Algebraic modelling is the study of relationships and the formulation of a mathematical rule or model to describe such relationships.

CHAPTER OUTLINE



IN THIS CHAPTER YOU WILL:

- graph linear functions of the form $y = mx + c$
- find the gradient and y-intercept of a line from its graph
- use linear functions to model practical situations and interpret the meaning of the gradient and y-intercept of the line in context
- graph and solve problems involving direct linear variation
- interpret and apply linear conversion graphs

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TERMINOLOGY

constant of variation
direct linear variation
linear function
rise
y-intercept
conversion graph
gradient
linear modelling
run
dependent variable
independent variable
proportional to
vertical intercept

WS
Assignment 7

SkillCheck

1 Graph this table of values on a number plane and rule a line through the points.

x	-1	0	1	2
y	-4	-1	2	5

2 Copy and complete each of these tables of values using the given formula.

a $y = 2x + 7$

x	-1	0	1	2
y				

b $y = -x - 4$

x	-1	0	1	2
y				

3 If $y = mx + c$, find the value of c if $x = 2$, $y = 5$ and $m = 3$.

4 If $y = kx$:

a find the value of y if $k = 0.7$ and $x = 4$

b find the value of k if $x = 2.5$ and $y = 10$.

5 Given the formula $V = -450t + 2575$:

a find the value of V if:

i $t = 3$ **ii** $t = 4.2$.

b find the value of t if:

i $V = 1450$ **ii** $V = 10$.

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7.01 Graphing linear functions

The formula or equation $y = 3x - 2$ is called a **linear function** because its graph is a straight line. **Linear** means 'of a line', while a **function** is an algebraic rule similar to the 'number machine' shown below, that changes an input value, x, into an output value, y.

Function: $y = 3x - 2$

$x = 5$ —input→ process —output→ $y = 13$

A line has a **gradient** (the measure of its steepness) and a **y-intercept** (the value where the line crosses the y-axis).

Drawing gradients

Graphing linear functions

A page of number planes

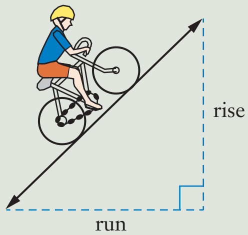
Graphing lines: graphics calculator

Graphing linear equations

Gradient of a line

The gradient of a line is given by the formula:

$$m = \frac{\text{rise } (\uparrow)}{\text{run } (\rightarrow)}$$



- The **rise** is the vertical change in position (going up).
- The **run** is the horizontal change in position (going across to the right).

EXAMPLE 1

a Copy and complete this table of values for the linear function $y = 3x - 2$.

x	−1	0	1	2	3
y					

b What pattern do you notice in the y values in the bottom row of the completed table?

c Graph $y = 3x - 2$ on a number plane.

d Find the gradient and y-intercept of the line you drew in part **c**.

Gradient and y-intercept

$y = mx + c$

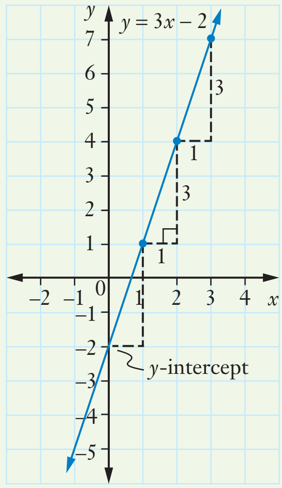
Solution

a

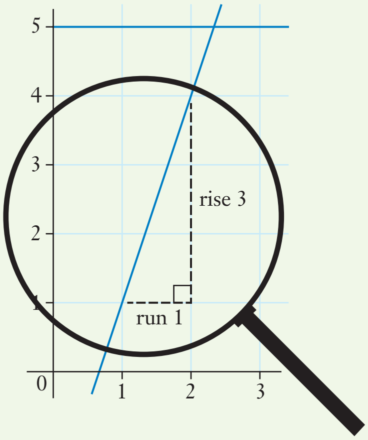
x	−1	0	1	2	3
y	−5	−2	1	4	7

b The y values increase by 3 each time.

c Use the (x, y) ordered pairs from the table to graph the points: (−1, −5), (0, −2), (1, 1), (2, 4), (3, 7). Then rule and label the line through the points.



d Choose two points on the line, say (1, 1) and (2, 4).



$$\text{Gradient } (m) = \frac{\text{rise}}{\text{run}} = \frac{3}{1} = 3$$

The y-intercept is −2.

Gradient and y-intercept of a line

Linear function

A **linear function** has the form $\boldsymbol{y = mx + c}$, where m is the gradient and c is the y-intercept.

$y = mx + c$ — gradient: m; y-intercept: c

$y = 3x - 2$ — gradient 3; y-intercept −2

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EXAMPLE 2

a Complete this table of values for the linear function $y = -2x + 4$.

x	–1	0	1	2	3
y					

b What pattern do you notice in the y values in the completed table?

c Graph $y = -2x + 4$ on a number plane.

d Find the gradient and y-intercept of the line you drew in part **c**.

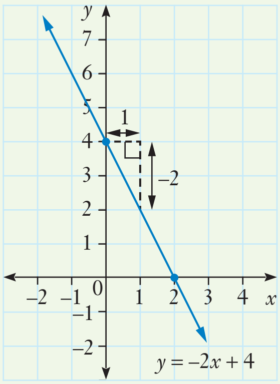
Solution

a

x	–1	0	1	2	3
y	6	4	2	0	–2

b The y values decrease by 2 each time.

c



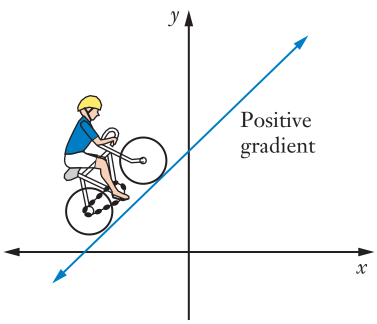
d Using points (0, 4) and (1, 2):

$$\text{Gradient } (m) = \frac{\text{rise}}{\text{run}} = \frac{-2}{1} = -2$$

The y-intercept is 4.

Positive and negative gradients

A line with a **positive gradient** slopes upwards (from left to right) because its y values are *increasing*. A gradient of 3 means that, for each 1 unit we move across, the line goes up 3 units or, as the x values increase by 1, the y values *increase* by 3.



Graph sloping upwards

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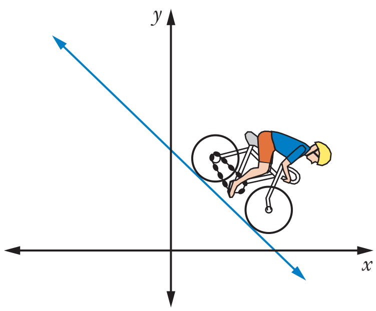
A line with a **negative gradient** slopes downwards (from left to right) because its y values are *decreasing*. A negative gradient has a 'negative rise', or a 'drop'. A gradient of –2 means that, for each unit we move across, the line goes down 2 units, or as the x values increase by 1, the y values *decrease* by 2.



Graph sloping downwards

EXAMPLE 3

Graph each linear function after finding its gradient and y-intercept.

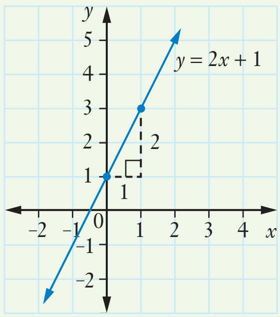
a $y = 2x + 1$ **b** $y = -4x$ **c** $y = \frac{2}{3}x - 4$

Solution

a Gradient = 2, y-intercept = 1

To graph the line:

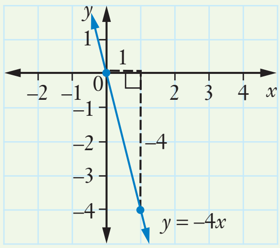
- plot the y-intercept, 1, on the y-axis
- indicate a gradient of 2 from this point, by moving across 1 unit and up 2 units and marking that point
- rule a line through both points you marked
- draw arrows at the ends and label the line with its equation.



b Gradient = –4, y-intercept = 0

To graph the line:

- plot the y-intercept, 0, at the origin
- indicate a gradient of –4 from this point, by moving across 1 unit and down 4 units and marking that point
- rule a line through both points you marked, add arrows and label.



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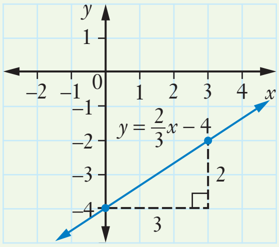
c Gradient $= \frac{2}{3}$, y-intercept $= -4$

To graph the line:

- plot the y-intercept -4 on the y-axis
- indicate a gradient of 3 from this point, by moving across 3 units and up 2 units and marking that point
- rule a line through both points you marked, add arrows and label.



Exercise 7.01 Graphing linear functions

A page of number planes

Number plane grid paper

1 Write the equation of the line with:

a gradient 3, y-intercept 7 **b** gradient –2, y-intercept 1

c gradient 1, y-intercept –1 **d** gradient $\frac{1}{3}$, y-intercept $\frac{1}{2}$

e gradient $-\frac{5}{4}$, y-intercept 0 **f** gradient 0, y-intercept 5

Example 1

Example 2

Example 3

2 Graph each linear function.

a $y = 4x - 3$ **b** $y = \frac{1}{2}x + 2$ **c** $y = x - 2$

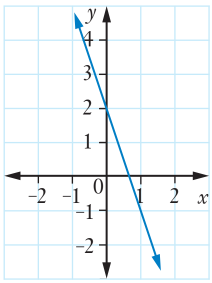
d $y = -2x$ **e** $y = 2x - 5$ **f** $y = \frac{3}{5}x - 1$

g $y = -x + 4$ **h** $y = \frac{4}{3}x + 1$ **i** $y = \frac{1}{4}x + 3$

3 What is the gradient of this line? Select **A**, **B**, **C** or **D**.

A $\frac{1}{3}$ **B** $-\frac{1}{3}$

C 3 **D** –3



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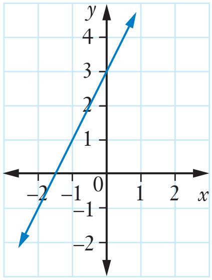
4 Find the gradient and y-intercept of each line below, and write its equation.

a



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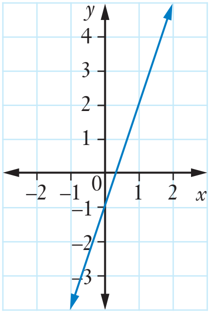
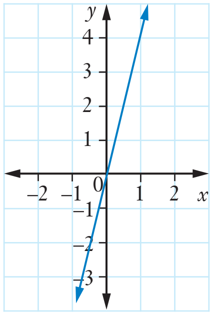


c



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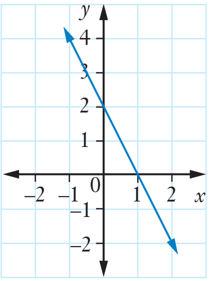

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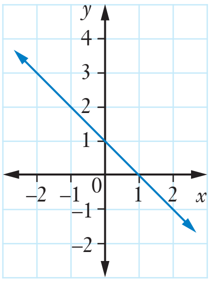

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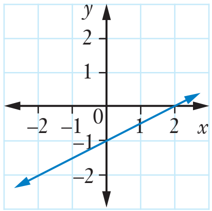

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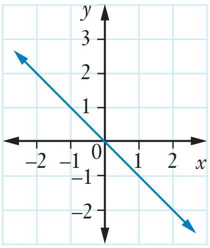

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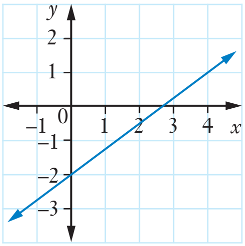

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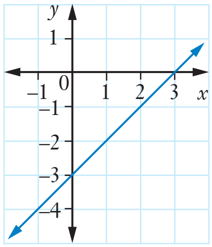

INVESTIGATION

GRADIENTS

a What does a line with a gradient of 0 look like?

b What does a line with a gradient of 1 look like?

c What does a line with a gradient of –1 look like?

d What is the highest gradient a line can have?

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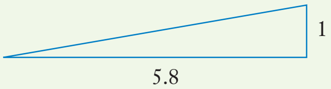
DID YOU KNOW?

Steep roads and hills

Australia is a fairly flat continent but in Europe road signs often display the steepness of a road for drivers (or the steepness of a hill for hikers).

Consider this example:



For this example, the steepness is $\frac{1}{5.8}$, also written as '1 : 5.8' or '1 in 5.8'.

Bulli Pass, north of Wollongong, has a gradient of $\frac{1}{6}$ while Victoria Pass, east of Lithgow, has a gradient of $\frac{1}{8}$. Which pass is steeper?

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A gradient of 8% (that is $\frac{8}{100}$) means a rise of 8 units for every 100 units of run.

7.02 The gradient formula

The formula for the gradient of a line is useful if:

- the coordinates of two points on the line are given, or
- a table of values is given.

Gradient formula

$$m = \frac{\text{rise} \ (\uparrow)}{\text{run} \ (\rightarrow)} = \frac{\text{change in } y}{\text{change in } x}$$

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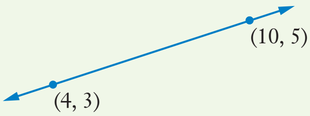
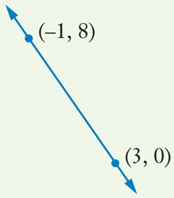
EXAMPLE 4

Find the gradient of each line.

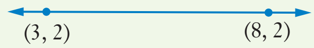
a



b



c



Solution

a $m = \dfrac{\text{change in } y}{\text{change in } x}$

$= \dfrac{5-3}{10-4}$

$= \dfrac{2}{6}$

$= \dfrac{1}{3}$ A gradient of $\dfrac{1}{3}$ is not steep.

b $m = \dfrac{0-8}{3-(-1)}$

$= \dfrac{-8}{4}$

$= -2$ Negative gradient, the line slopes downward

c $m = \dfrac{2-2}{8-3}$

$= \dfrac{0}{5}$

$= 0$ Zero gradient line is flat

EXAMPLE 5

Find the gradient of the line represented by each table of values.

a

x	0	4	8	20
y	5	11	17	35

b

x	2	6	9	12
y	4	–8	–17	–26

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Solution

a Choose any two points from the table, say (0, 5) and (4, 11):

$$m = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{11 - 5}{4 - 0}$$

$$= \frac{6}{4}$$

$$= \frac{3}{2}$$

The gradient is $\frac{3}{2}$.

b Using (2, 4) and (6, –8) from the table:

$$m = \frac{-8 - 4}{6 - 2}$$

$$= \frac{-12}{4}$$

$$= -3$$

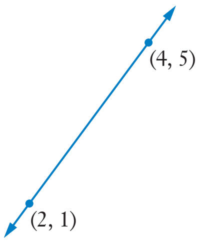
Negative gradient (y decreases as x increases)

Exercise 7.02 The gradient formula

Example 4

1 Find the gradient of each line.

a



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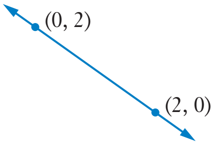

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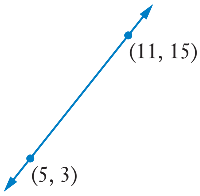

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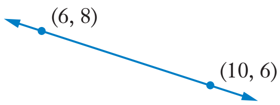

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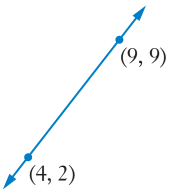

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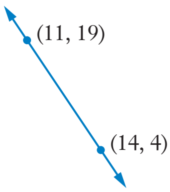

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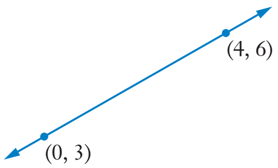

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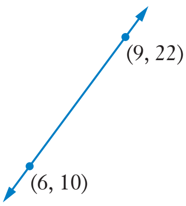

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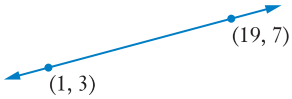

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Example 5

2 Find the gradient of the line represented by each table of values.

a

x	0	4	7	10
y	–1	11	20	29

b

x	1	5	7	12
y	1	21	31	56

c

x	0	8	12	20
y	3	5	6	8

d

x	1	3	6	8
y	7	3	–3	–7

e

x	0	10	25	40
y	–4	2	11	20

f

x	2	8	14	16
y	4	1	–2	–3

7.03 Linear modelling

WS Finding the equation of a line

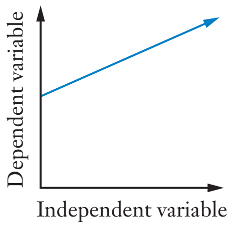
PS Linear functions code puzzle

Scientists and researchers observing patterns in nature and society often use a mathematical formula to represent a real-life situation. This is called **algebraic modelling** and if the relationship uses the linear function $y = mx + c$, it is called **linear modelling**.

Dependent and independent variables

Because the value of y depends on the value of x, y is called the **dependent variable** and x is called the **independent variable**.

When a function is graphed, the independent variable is shown on the horizontal axis, while the dependent variable is shown on the vertical axis.



Interpreting the gradient and the y-intercept

The **gradient** measures the steepness of a line, but it also shows how quickly the y-values are changing.

Look at this table of values for $y = 5x – 4$:

+1 +1 +1 +1

x	–1	0	1	2	3
y	–9	–4	1	6	11

+5 +5 +5 +5

As the x values increase by 1, the y values increase by 5.
If this linear function is graphed, for every 'run' of 1 unit, there is a 'rise' of 5 units, giving a gradient of $\frac{5}{1} = 5$.

The gradient and y-intercept

- The **gradient** of a linear function is the **rate of change** of y.
- The higher the gradient, the steeper the line, and the faster y increases relative to x.
- The **y-intercept** is the value of y when $x = 0$.

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EXAMPLE 6

Linear modelling

Brett is organising a 21st birthday party. The total cost of the party will include \$200 for hiring the hall, \$120 for the music and \$14 per person for catering. The table below shows the total cost, \$$C$, for different numbers of guests, n, attending the party.

Number of guests, n	80	100	120	130	150
Party cost, \$$C$	1440	1720	2000	2140	2420

a Which variable, n or C, is the dependent variable?

b Find the gradient of the linear relationship between n and C.

c Find the vertical intercept of this linear relationship.

d Write the linear function for C in terms of n.

e If this function was graphed, which variable would be shown on the horizontal axis?

f What does the vertical intercept of this function represent?

g Find the cost of the party if there were 95 guests.

Vertical intercept is the more general name for the y-intercept, because linear functions can use variables other than x and y.

Solution

a C, because C depends on n.

b Choosing the ordered pairs (80, 1440) and (100, 1720) from the table:

$$m = \frac{1720 - 1440}{100 - 80} = \frac{280}{20} = 14$$

The gradient is 14.

c The linear function is: $C = mn + c$

The gradient is: $m = 14$ — From the answer to part **b**

So: $C = 14n + c$

To find the value of c, the vertical intercept, substitute an ordered pair into the function.

Substitute (100, 1720):

$1720 = 14(100) + c$

$1720 = 1400 + c$

$c = 320$

The vertical intercept is 320.

d The function is $C = 14n + 320$.

This formula should work for the other ordered pairs in the table.

e n, because it is the independent variable.

f The vertical intercept, 320, is the value of C when $n = 0$, which is the cost of the party if there are no guests attending. This makes sense because it is the total fixed cost of hiring the hall and the music (\$200 + \$120).

g Substitute $n = 95$ into the function:

$$\begin{aligned} C &= 14n + 320 \\ &= 14(95) + 320 \\ &= 1650 \end{aligned}$$

The cost for the party with 95 guests is \$1650.

EXAMPLE 7

A criminologist discovered that the number of crimes committed per month, C, in a big city decreased as the number of police officers, P, patrolling the city increased. After graphing her data on a number plane, she found the linear relationship to be $C = -3P + 3250$.

a What is the independent variable in this relationship?

b Copy and complete this table for the equation $C = -3P + 3250$.

Number of police, *P*	50	150	200	250	300
Crimes per month, *C*					

c Graph $C = -3P + 3250$ on the number plane.

d What is the gradient of the graph you drew in part **c** and what does it represent?

e What is the vertical intercept of the graph you drew in part **c** and what does it represent?

f Calculate how many crimes are committed when 100 police officers are on patrol.

g Calculate how many police officers are required to reduce the number of crimes to 1900.

Solution

a P, because C depends on P.

b

Number of police, *P*	50	150	200	250	300
Crimes per month, *C*	3100	2800	2650	2500	2350

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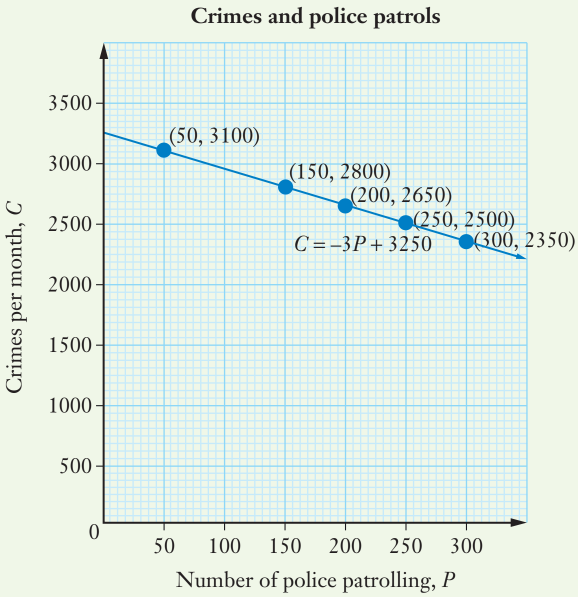
c



d From the equation, the gradient is –3 and it represents the rate of change in crime as the number of police increases. As the number of police increases by 1, the number of crimes decreases by 3.

e The vertical intercept is 3250, and this represents the number of crimes if no police officers were on patrol. It is the value of C when $P = 0$.

f Substitute $P = 100$:

$$C = -3P + 3250$$
$$C = -3(100) + 3250$$
$$= 2950$$

2950 crimes are committed when 100 police are on patrol.

g Substitute $C = 1900$:

$$C = -3P + 3250$$
$$1900 = -3P + 3250$$
$$-3P = -1350$$
$$P = \frac{-1350}{-3}$$
$$= 450$$

450 police officers are required to reduce the number of crimes to 1900.

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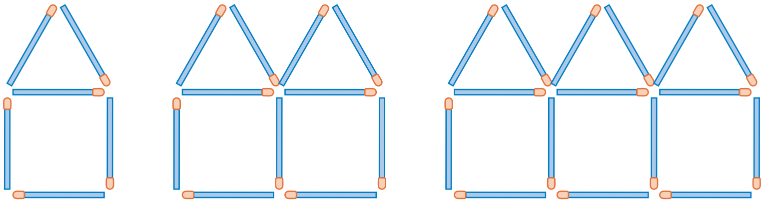
Exercise 7.03 Linear modelling

Example 6

1 This is a matchstick pattern of houses.



One house uses 6 matches.

Two houses use 11 matches.

Three houses use 16 matches.

a Copy and complete this table for the pattern above.

Number of houses, *h*	1	2	3	4	5	6
Number of matches, *N*						

b What is the dependent variable?

c Find the linear relationship in the form $N = mh + c$.

d How many matches are required to make 20 houses?

e How many houses can be made from 81 matches?

f Graph the linear function on a number plane.

g Write the gradient and the vertical intercept of the graphed line.

2 This table shows the cost, C cents, of mobile phone calls under the Oz-Zone Budget Plan, for calls of different lengths, t minutes.

Length of call, *t* (min)	1	2	5	10	15
Cost, *C* (cents)	102	182	422	822	1222

a Find the linear relationship in the form $C = mt + c$.

b If this function was graphed on a number plane, which variable would be shown on the vertical axis?

c Use the relationship you found in part **a** to calculate the cost of an 18-minute call.

d What is the vertical intercept of this function and what does it represent?

e If a phone call is extended by 3 minutes, by how much would its cost increase?

f How long is a phone call under this plan if it cost \$5.82?

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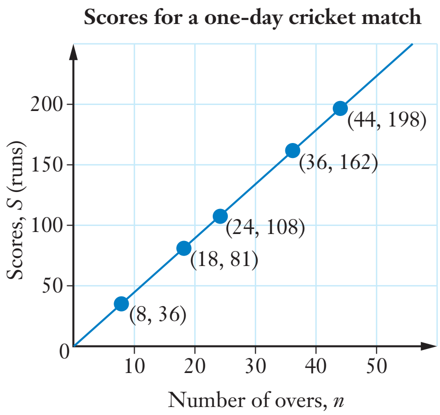
3 A cricket team's progressive score during a one-day cricket match can be approximated by the linear function graphed here.
A one-day match has 50 overs, where an over is a set of 6 balls bowled by the same bowler. The variable, n, represents the number of overs bowled, while S represents the total number of runs scored by the batting team.



- **a** Is S the dependent variable or the independent variable?
- **b** Find the formula for S in terms of n.
- **c** The gradient of this linear function is also the team's run rate. What is the gradient of the function and in what units is this run rate measured?
- **d** What is the vertical intercept and what does it represent?
- **e** What was the score after:
  - **i** the 21st over?
  - **ii** the 50th over?
- **f** At the end of which over had the score reached:
  - **i** 54 runs?
  - **ii** 180 runs?
- **g** The graph for a real cricket match would not be a straight line but would normally flatten out later in the innings. Why?

4 The value of a notebook computer depreciates according to the formula $V = -420t + 1900$, where V is the value in dollars and t is the time in years.

Example 7

- **a** Copy and complete this table for the formula $V = -420t + 1900$.

Time, t (years)	1	2	3	4
Value of computer, V ($)				

- **b** Graph this linear relationship on a number plane.
- **c** What does the gradient of this linear function represent?
- **d** What was the value of the computer after $2\frac{1}{2}$ years?
- **e** What was the original value of the computer?
- **f** This linear model does not work when $t = 5$ and beyond. Why not?
- **g** Find, correct to one decimal place, the time when the computer has zero value.

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5 Yasmin works for a pizza shop and, each day, she earns a base pay of \$50, plus \$3 for every pizza she delivers.

a If n is the number of pizzas Yasmin delivers in a day and P is her total pay in dollars, write a formula for P in terms of n.

b If this linear function was graphed, which variable would be represented on the horizontal axis?

c What is the vertical intercept of this function and what does it represent?

d How much will Yasmin earn for delivering 28 pizzas in a day?

e If Yasmin earned \$98 today, how many pizzas did she deliver?

6 This table shows the linear relationship between distances measured in miles and distances measured in kilometres.

Miles, *M*	15	25	30	45
Kilometres, *K*	24	40	48	72

a Graph this linear relationship on a number plane.

b Find the equation of the line.

c What can you say about the value of the vertical intercept? Why?

d What is the gradient and what does it represent?

e Use your equation from part **b** to convert:

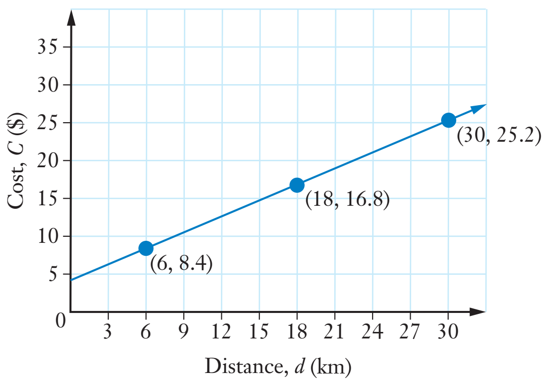
i 100 miles to kilometres **ii** 100 km to miles.

f Use your graph from part **a** to convert:

i 12 miles to kilometres **ii** 20 kilometres to miles.

7 This graph shows the linear relationship between the distance, d km, travelled by a truck and the running costs, \$$C$, for the trip.

Running costs of a truck



a What is the dependent variable?

b Find the gradient and the vertical intercept of this linear relationship.

c Write the formula for this linear relationship.

d If the length of a trip is extended by 5 km, by how much will the charge increase?

e Calculate the running costs for a trip of length:

i 20 km **ii** 0 km.

f Calculate the distance travelled if the running costs were \$37.80.

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8 During summer, crickets chirp faster at night if the temperature is higher. There is a linear relationship between the temperature and a cricket's chirping rate, shown in the table below.

Temperature, T (°C)	12	15	19	22	28
Chirp rate, n (chirps/min)	72	96	128	152	200

a Is T the dependent variable or the independent variable?

b Find the linear function for n in terms of T.

c Graph the linear function you found in part **b**.

d If the temperature increases by 2°C, what happens to the crickets' chirp rate?

e Find the chirp rate of a cricket when the temperature is 26°C.

f At what temperature does a cricket chirp 144 times per minute?

g What is the vertical intercept of this function? Why doesn't the linear model work for this value?

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7.04 Direct linear variation

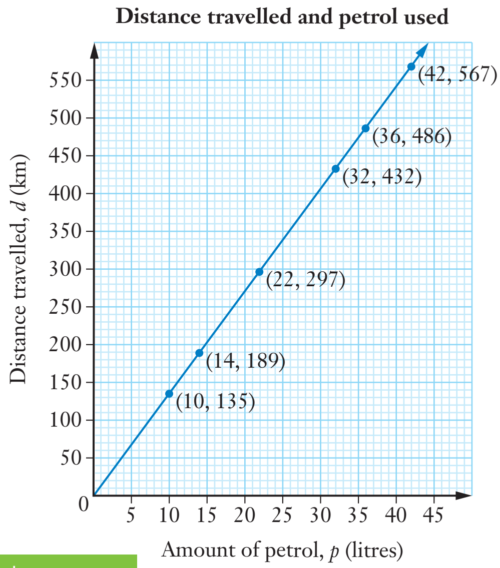
Ann-Marie noticed that, with her new car, she could drive a distance of 567 km on a full tank of petrol (42 litres). She also discovered the results shown in the table below.

Amount of petrol used, p (L)	10	14	22	32	36	42
Distance travelled, d (km)	135	189	297	432	486	567

When she graphed these values, Anne-Marie found a linear relationship.

This relationship must be of the form $d = mp + c$, but the vertical intercept is 0 because, when $p = 0$, $d = 0$ (no petrol, no distance). So the linear function is $d = mp$.

To find the value of m, substitute any point from the table into the equation, for example (10, 135).



$$d = mp$$

$$135 = m\,(10)$$

$$m = 10$$

$$= 13.5$$

$\therefore$ The linear function is $d = 13.5p$.

Choose another point from the table to check that this formula is correct.

Because $d = 13.5p$, the distance, d, is found by multiplying the amount of petrol, p, by a constant amount, 13.5. This is an example of **direct linear variation**, and we can say that 'd varies directly as p', or that 'd is directly **proportional to** p'.

Direct linear variation

If y **varies as** x, or y is **directly proportional to** x, then $y = kx$, where k is a constant.
k is called the **constant of variation** or **constant of proportionality**.

A direct linear relationship exists between x and y. If x increases (or decreases), y increases (or decreases). If x is doubled (or halved), y is doubled (or halved).

EXAMPLE 8

The mass, M (in kilograms), of a metal varies directly as its volume, V (in cubic centimetres).

Volume, V (cm^3)	0	48	60	80	116	140
Mass, M (kg)	0	14.4	18	24	34.8	42

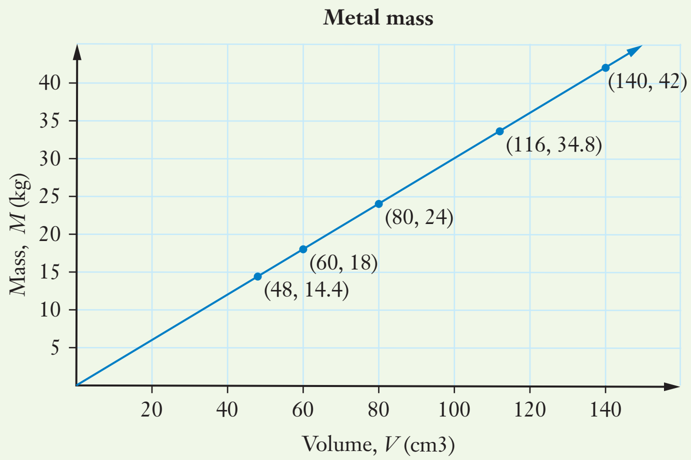
a Graph the relationship between M and V.

b Find the equation for M in terms of V.

c Find the mass of 212 cm^3 of the metal.

Solution

a



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b $M = kV$

Substituting (60, 18) from the table to find k:

$$18 = k(60)$$
$$k = \frac{18}{60}$$
$$= \frac{3}{10}$$
$$\therefore M = \frac{3V}{10} \quad (\text{or } M = 0.3V)$$

Choose another point from the table to check that this formula is correct.

c Substitute $V = 212$ into the formula.

$$M = \frac{3(212)}{10}$$
$$= 63.6$$

The mass of the metal is 63.6 kg.

EXAMPLE 9

The stretch, S (in centimetres), of a spring varies as the mass, M (in kilograms), of the load pulling it. A load of 24 kg causes the spring to stretch 15 cm.

a Find the variation equation relating S to M.

b What is the stretch caused by a load of 13 kg?

c What is the mass of the load that will cause a stretch of 25 cm?

d What is a limitation of this linear model?

Solution

a $S = kM$

Substitute $M = 24$ and $S = 15$ to find k.

$$15 = k(24)$$
$$k = \frac{15}{24}$$
$$= \frac{5}{8} \quad (\text{or } 0.625)$$
$$\therefore S = \frac{5M}{8} \quad (\text{or } S = 0.625M)$$

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b When $M = 13$:

$$S = \frac{5(13)}{8}$$
$$= 8.125$$

A stretch of 8.125 cm is caused by a load of 13 kg.

c When $S = 25$:

$$25 = \frac{5M}{8}$$
$$5M = 200$$
$$M = 40$$

A 40 kg load will cause a stretch of 25 cm.

d For a very heavy load, the spring will become permanently stretched and will not return to its original length when the load is removed.

Linear variation problems

To solve a linear variation problem:

1. Identify the two variables (say x and y) and form a variation equation, $y = kx$.
2. Substitute values for x and y to find k, the constant of variation.
3. Rewrite $y = kx$ using the value of k.
4. Substitute a value for x or y into $y = kx$ to solve the problem.

Exercise 7.04 Direct linear variation

Example 8

1 Copy and complete this table after finding the value of k in $y = kx$.

x	1	4	6	10	11	15
y		14	21			52.5

Example 9

2 The distance travelled by a bicycle varies directly with the number of revolutions made by the pedals.

a Form a variation equation and find the constant of variation, given that the bicycle travels 55 metres for 20 revolutions of the pedals.

b Calculate the distance travelled for 33 revolutions of the pedals.

c Calculate the number of revolutions of the pedals required for the bike to travel 99 metres.

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3 The increase in pressure experienced by a scuba diver is directly proportional to her depth under the water. The increase in pressure at 25 m is 147 kilopascals (kPa).

a Form a variation equation.

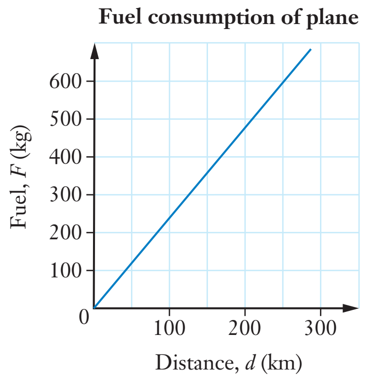
b What increase in pressure is experienced at 40 m?

c At what depth is the increase in pressure 833 kPa?

4 The graph illustrates the fact that the mass of fuel an aeroplane requires varies directly with the distance it flies.

a Form a variation equation, using a point on the graph to find the constant of variation.

b Use the equation to calculate the amount of fuel required to fly 3250 km.



5 A speed expressed in km/h is directly proportional to the same speed expressed in miles/h.

Speed in miles/h, x	30	36	45	50	62.5	74
Speed in km/h, y	48	57.6	72	80	100	118.4

a Find the equation for this table of values.

b Convert 60 miles/h to km/h.

c Convert 120 km/h to miles/h.

6 For an object that is cooling, the drop in temperature varies directly with time. If the temperature drops 8°C in 5 minutes, which of the following is the amount of time it would take for the temperature to drop 10°C? Select **A**, **B**, **C** or **D**.

A 6.25 min **B** 7 min **C** 12.8 min **D** 16 min

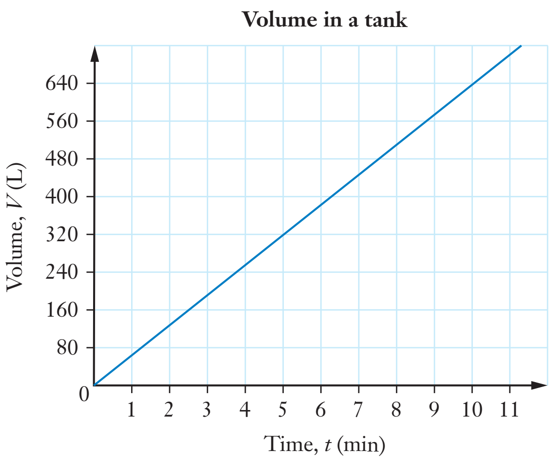
7 The volume, V litres, of water in a tank as it is filled is proportional to the time taken t minutes, as shown on the graph.

a Find the constant of variation.

b What does the constant you found in part **a** represent?

c Calculate the amount of water in the tank after 8 minutes.

d Calculate how long it takes to pump 960 L of water into the tank.



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8 The distance travelled by a marathon runner varies directly with time.

a If he covers 9.75 km in 45 minutes, calculate how long it will take him to run 26 km.

b What is a limitation of this linear model?

9 The weight of an astronaut on Mars is proportional to his weight on Earth. A 72 kg astronaut weighs 27.4 kg on Mars.

a Calculate how much a 60 kg astronaut weighs on Mars, correct to one decimal place.

b If an astronaut weighs 32 kg on Mars, calculate his weight on Earth, correct to one decimal place.

10 Nick noticed that, during his road trip, the amount of petrol used by his car varied with the amount of time he drove. It consumed 45 L of petrol in 4 hours. How long, to the nearest minute, will Nick's car take to consume 100 L of petrol?

11 The download time of a computer file is directly proportional to the size of the file. If a file of 1800 kilobytes requires 36 seconds to download, calculate:

a how long it will take to download a 3000 kilobyte file

b the size of a file that requires 80 seconds to download.

12 The compression of a car spring is proportional to the force applied. A force of 200 N (newtons) causes a compression of 1.6 cm. Which of the following amounts of compression is caused by a 5000 N force? Select **A**, **B**, **C** or **D**.

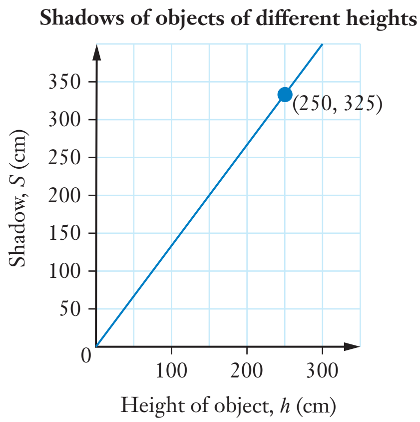
A 25 cm **B** 40 cm **C** 62.5 cm **D** 200 cm

13 At 3:30 p.m. one day, the shadows of objects of different heights were measured, then graphed.

a Find the equation of the line.

b Calculate the length of the shadow of a lamppost of height 244 cm.

c What is the height of a letterbox, correct to the nearest centimetre, if its shadow is 120 cm long?



14 When an object is dropped under gravity, its speed varies with time. Its speed after 5 seconds is 49 m/s.

a Find its speed after 12 seconds.

b Calculate the time taken to reach a speed of 175 m/s (to the nearest second).

c What is a limitation of this linear model?

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7.05 Conversion graphs

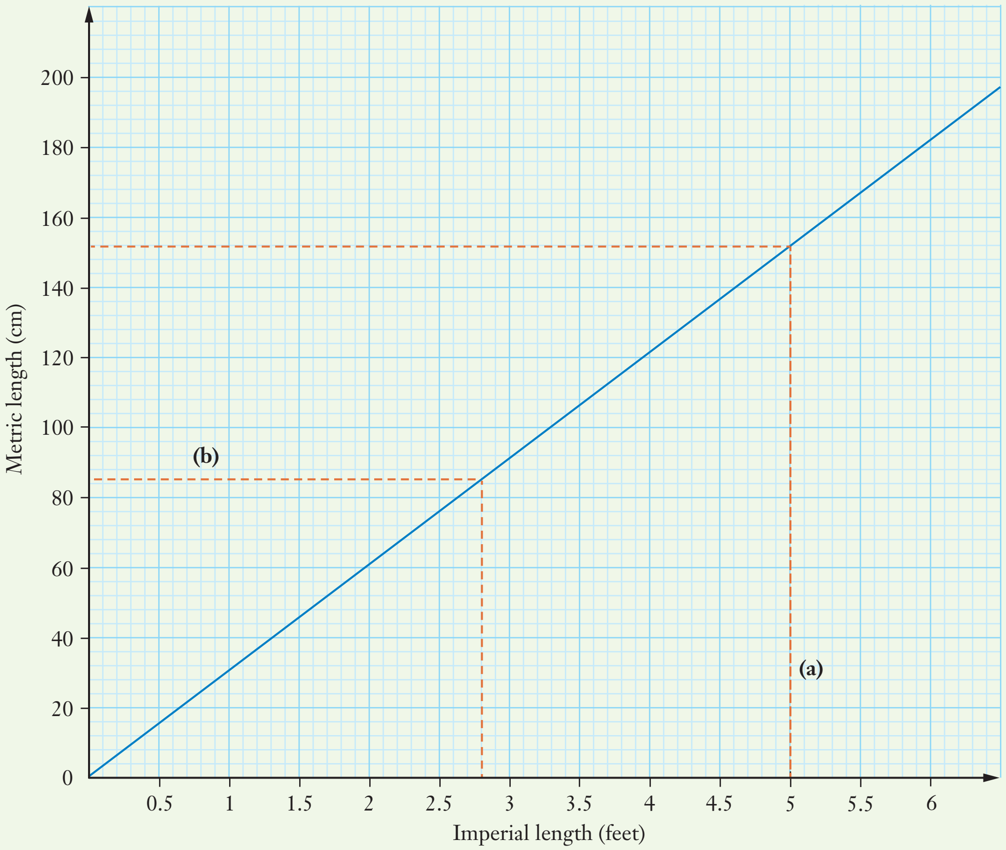
A **conversion graph** is used to convert between different units, such as between metric and imperial units, or between currencies in foreign currency exchange.

WS
Currency conversion graph

EXAMPLE 10

This conversion graph converts between feet (an imperial unit) and centimetres (a metric unit).

Converting feet to centimetres



a Kylie is 5 feet tall. What is her height in centimetres?

b A doorway is 85 cm wide. Convert this to feet, correct to one decimal place.

c The gradient of the line in the conversion graph is 30.5. What does this value represent?

Solution

a Reading from the graph: 5 feet = 152 cm

b Reading from the graph: 85 cm = 2.8 feet

c The gradient represents the rate of change of centimetres per foot: 30.5 cm = 1 foot

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Exercise 7.05 Conversion graphs

Example **10**

1 Use the conversion graph from Example **10** to answer the following questions.

a A bookshelf is 3.5 feet tall. What is its height in centimetres?

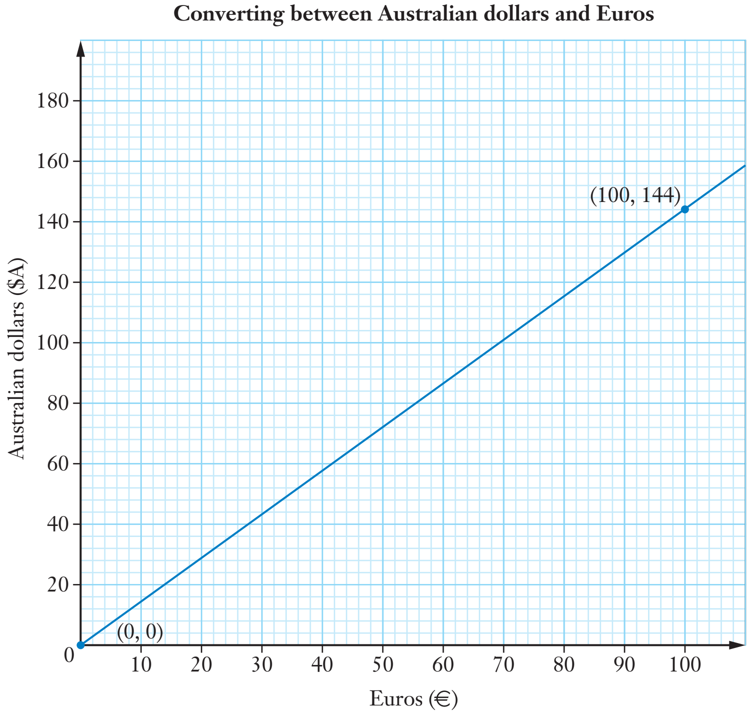
b Michael is 180 cm tall. Convert this height to feet, correct to one decimal place.

c Convert each length to centimetres.

i 2 feet **ii** 2.5 feet **iii** 6 feet

d Convert your own height to feet and inches, given that 1 foot = 12 inches.

2 This currency conversion graph converts between the euro (€) and the Australian dollar (A$).



Convert each European price to Australian dollars.

a a smart watch €88

b theme park entry €36

c a mobile phone €76

d a camera €52

e a city tour €25

f car hire per day €42

3 The conversion graph from Question **2** can also be used to convert small amounts. For example, from the graph: €46 = A$66. So it is also true that €0.46 = A$0.66.

Use the graph to help you convert each of the following prices to Australian dollars.

a a can of drink €1.20

b posting a letter €0.80

c 30 min of Internet use €3.40

d a newspaper €1.25

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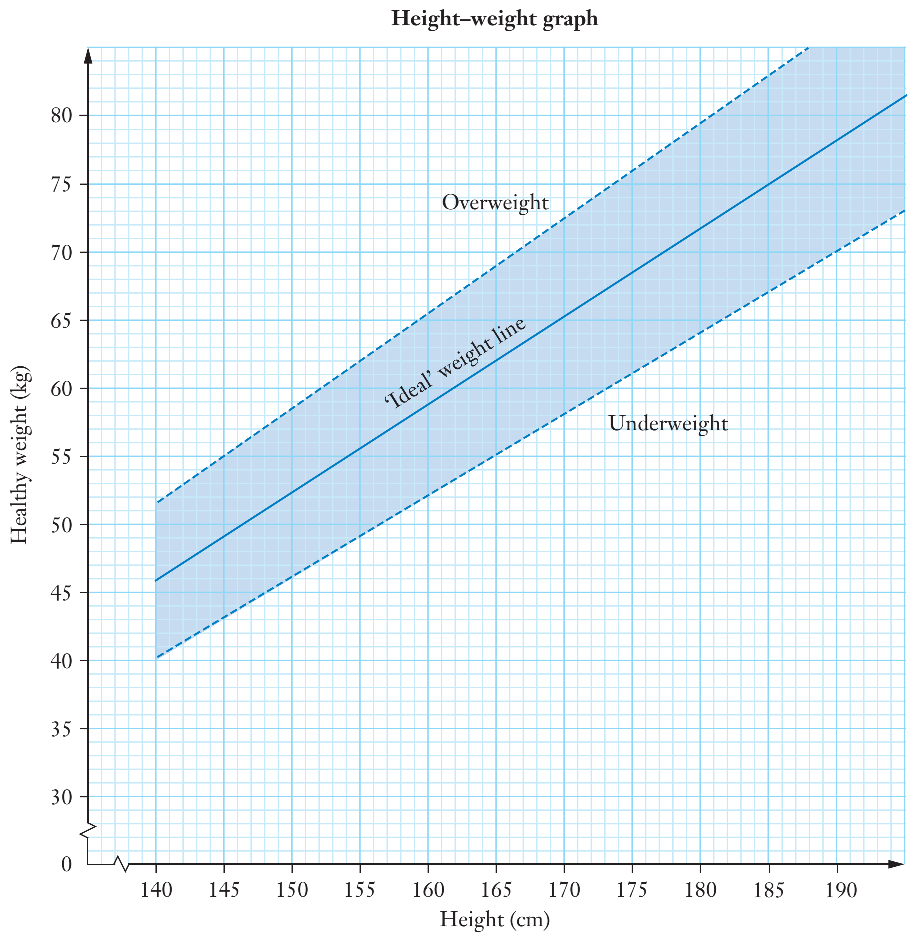
4 Use the conversion graph from Question **2** to convert each of the following Australian prices to euros.

a a movie DVD A$24
b a cordless phone A$76
c a soccer ball A$32
d a computer desk A$130
e an exercise bike A$108
f fish and chips A$7.50

5 Calculate the gradient of the currency conversion graph in Question **2**, correct to two decimal places. What does this value represent?

6 This height–weight graph shows the healthy weight range (shaded) for Australian adults of different heights.



a Find the 'ideal' weight of a person whose height is:
i 180 cm **ii** 155 cm **iii** 192 cm.

b Find the height of a person with an 'ideal' weight of:
i 72 kg **ii** 58 kg **iii** 80 kg.

c Calculate the gradient of the 'ideal' weight line. What rate does this gradient value represent?

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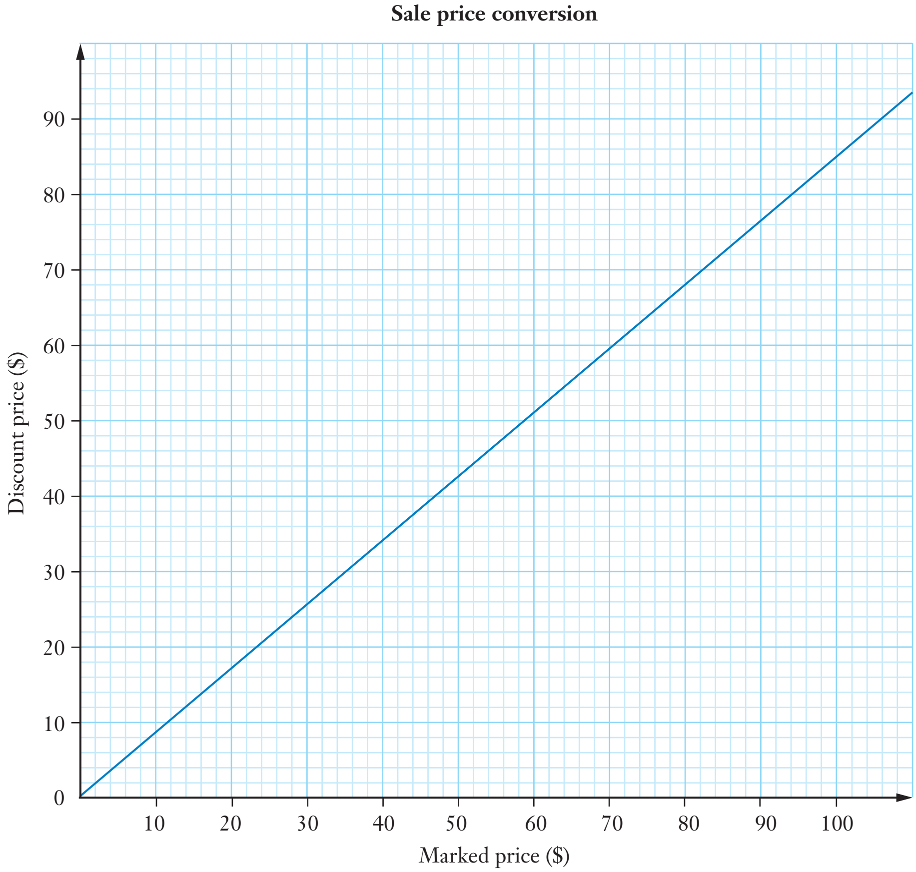
7 This graph converts marked prices to discount prices for a sale at Angry Andy's Bargain Basement.



a Find the discount price of an item with a marked price of:

i \$60   **ii** \$28   **iii** \$82.

b Find the marked price of an item with a discount price of:

i \$25   **ii** \$80   **iii** \$54.

c Calculate the gradient of this graph. What does the gradient value represent?

d Which of the following is the percentage discount given at Angry Andy's Bargain Basement? Select **A**, **B**, **C** or **D**.

A 5%   **B** 10%   **C** 15%   **D** 85%

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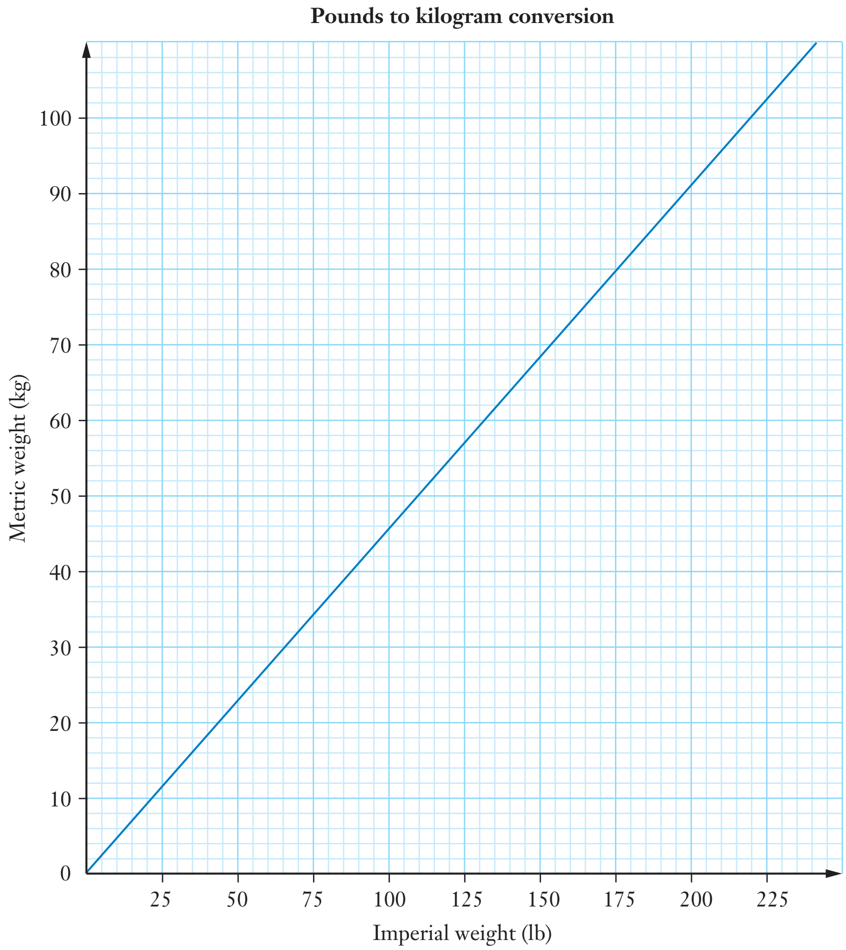
8 The graph below converts between pounds (an imperial unit of mass, with abbreviation 'lb') to kilograms (the metric unit).



a Convert each of these masses to kilograms.

i 90 lb **ii** 135 lb **iii** 160 lb

b Convert each of these masses to pounds.

i 40 kg **ii** 64 kg **iii** 90 kg

c What is the conversion rate in kg/lb?

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SAMPLE HSC PROBLEM

Mr and Mrs Dillon give their children weekly pocket money according to a function based on the age of the child, as shown in the table below.

Age, n years	7	9	10	13	15	18
Pocket money, $\$P$	4	12	16	28	36	48

a Find the formula for P in terms of n.

b What is the gradient for this function and what does it stand for?

c What is the pocket money for a child aged 16?

d What is the age of the child who receives \$20 per week?

e According to Mr and Mrs Dillon, this linear model holds true only for values of n from 7 to 18. Why do you think the formula is not valid for values of n:

i below 7? **ii** above 18?

Study tip

Attacking your weak areas

Most of your study time should be spent on attacking your weak areas to fill in any gaps in your Maths knowledge. Don't spend too much time on work you already know well, unless you need a confidence boost! Ask your teacher, use study guides or other textbooks to improve the understanding of your weak areas and to practise Maths skills. Use your topic summaries for *general revision*, but spend longer study periods on overcoming any difficulties in your mastery of the course.

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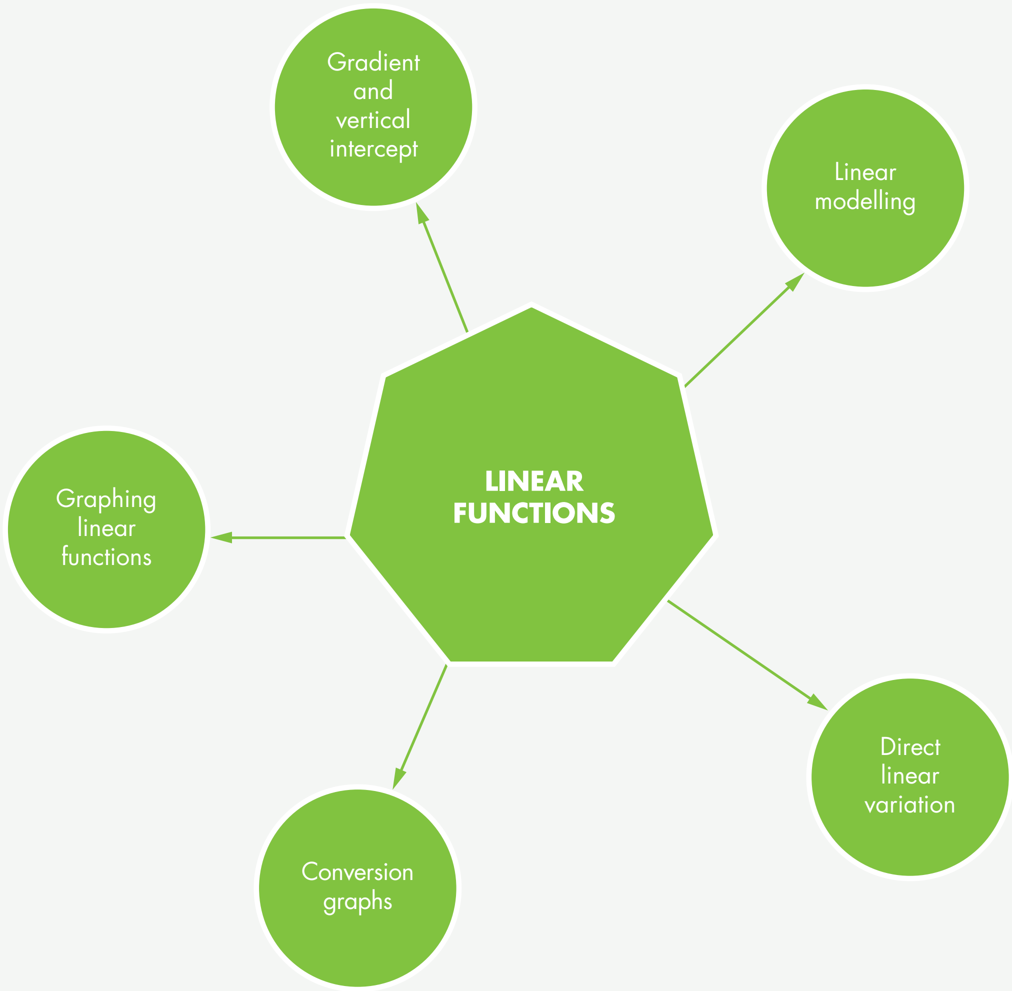
7. CHAPTER SUMMARY

This chapter, Linear functions, introduced the concept of the linear function and its graph and applications. Learn the meanings of the gradient and vertical intercept of a linear function and practise solving problems involving linear modelling, variation and conversion graphs.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map is shown below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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7. TEST YOURSELF

Exercise 7.01

1 Graph each of these linear functions on a number plane.

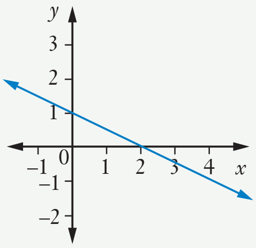
a $y = -2x + 1$

b $y = 4x$

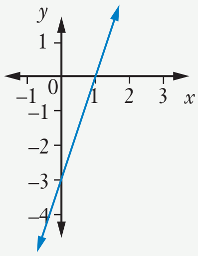
Exercise 7.01

2 Find the equation of each of these lines.

a



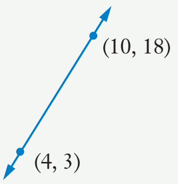
b



Exercise 7.02

3 Find the gradient of each of these lines.

a



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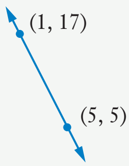

Exercise 7.02

4 Find the gradient of the function for each of these tables of values.

a

x	0	4	6	12
y	12	14	15	18

b

x	2	5	9	11
y	42	30	14	6

Exercise 7.03

5 The length, L inches, of a shoe has a linear relationship with the size, S, of the shoe.

Shoe size, S	2	5	7	8	12
Length, L (inches)	9	10	$10\frac{2}{3}$	11	$12\frac{1}{3}$

a Which is the independent variable?

b Graph this linear relationship.

c What is the gradient, and what does this represent?

d How long is a shoe of size 0? How is this shown on the graph?

e What is the formula for L in terms of S?

f What is the length of a shoe of size $7\frac{1}{2}$?

g What size is a shoe of length 13 inches?

Exercise 7.04

6 The distance travelled by a car is directly proportional to the number of rotations of its tyres. If 950 metres are travelled after 540 rotations, calculate how much distance, to the nearest kilometre, is covered after 10 000 rotations.

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7 This currency conversion graph converts between Thai baht (the currency of Thailand) and Australian dollars (A$).

Exercise 7.05

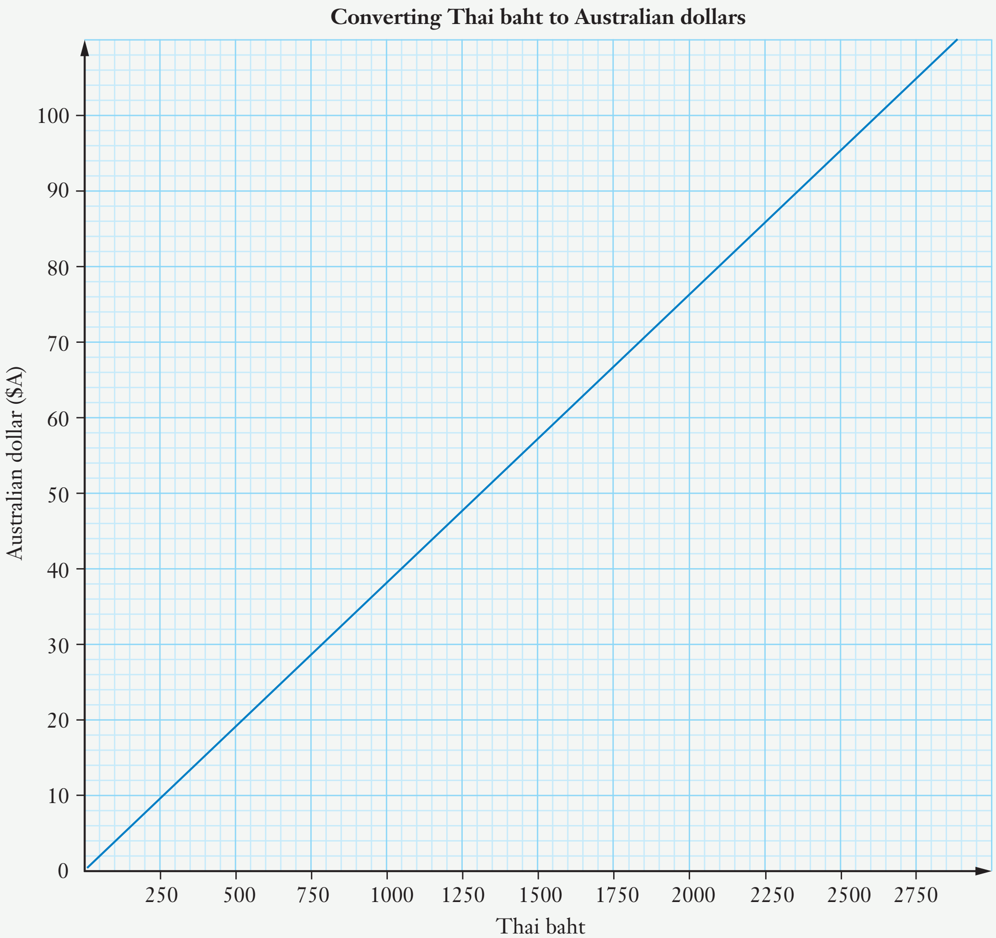
a Convert each of these Thai prices to Australian dollars (to the nearest dollar).

i 800 baht **ii** 2450 baht

b Convert each of these Australian prices to Thai baht (to the nearest 50 baht).

i A$65 **ii** A$82

c Calculate the gradient of this graph, correct to two decimal places. What does this value represent?



Chapter quiz