STATISTICAL ANALYSIS

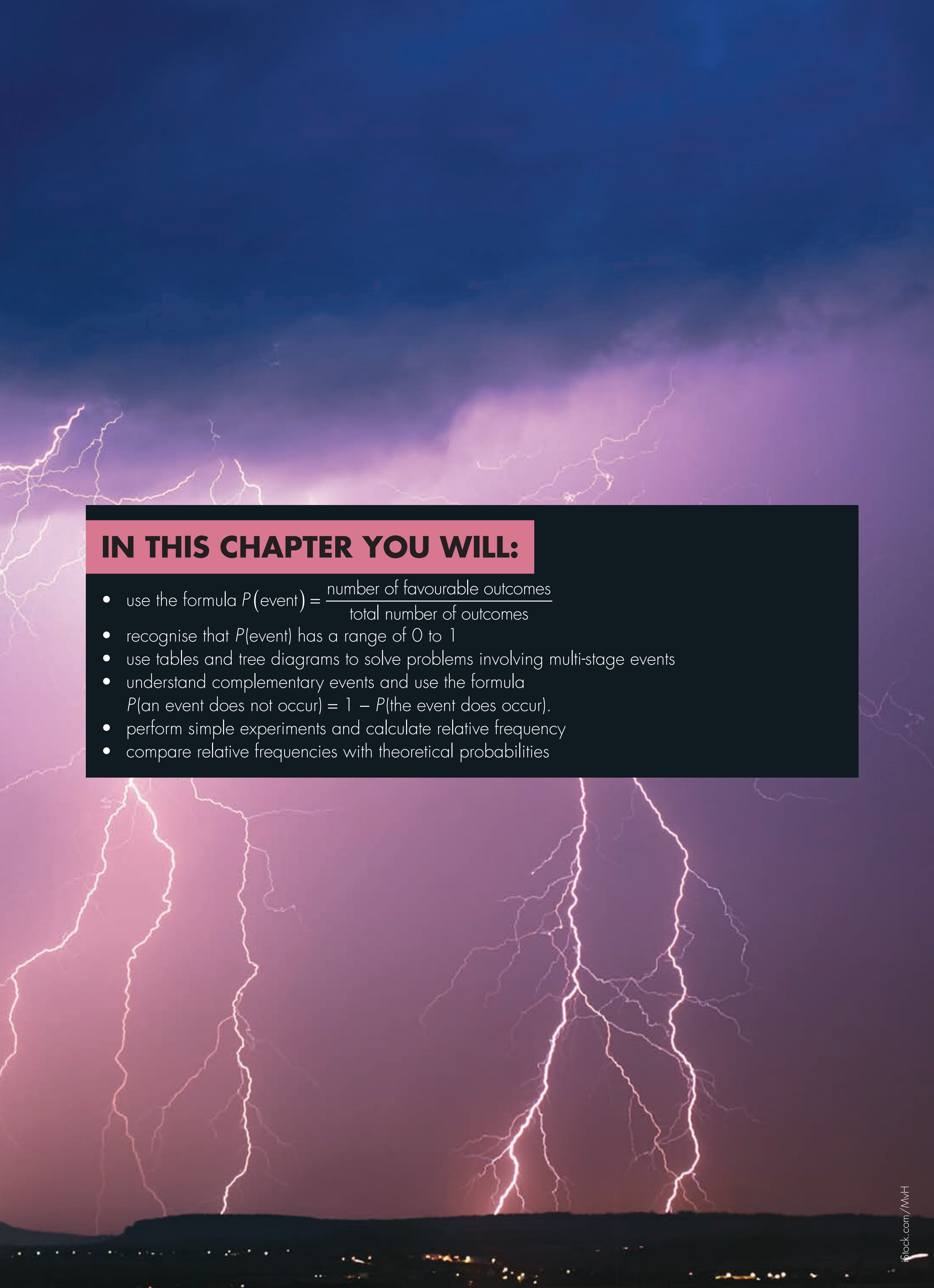
4.

PROBABILITY

Insurance originated in ancient Babylon and China around 3000 BCE, when voyagers and traders formed financial societies to insure their ships against damage or loss. In 600 CE, the Greeks and Romans organised similar schemes to help families during times of illness and to pay funeral expenses upon death. In 1693, the English astronomer and mathematician Edmund Halley, who discovered Halley's Comet, calculated the chances of people dying at different ages. Modern insurance companies use statisticians called actuaries to study the chances of losses and deaths occurring, such as:

- dying in a plane crash — 1 chance in 720 000
- being struck by lightning — 1 chance in 200 000
- dying in a car crash — 1 chance in 5000
- being in a house fire — 1 chance in 800
- going to prison — Male: 1 chance in 800. Female: 1 chance in 24 000.

CHAPTER OUTLINE



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IN THIS CHAPTER YOU WILL:

- use the formula $P(\text{event}) = \dfrac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$
- recognise that P(event) has a range of 0 to 1
- use tables and tree diagrams to solve problems involving multi-stage events
- understand complementary events and use the formula
  P(an event does not occur) = 1 − P(the event does occur).
- perform simple experiments and calculate relative frequency
- compare relative frequencies with theoretical probabilities

TERMINOLOGY

at least
chance
event
likely
probability
sample space
calculated probability
complementary event
frequency
multi-stage event
probability tree digram
tree diagram
certain
equally likely
impossible
outcome
relative frequency
unlikely

WS
Assignment 4

SkillCheck

1 Simplify each fraction.

a $\frac{28}{100}$ **b** $\frac{7}{77}$ **c** $\frac{13}{52}$ **d** $\frac{24}{36}$

2 Evaluate:

a $1-\frac{13}{40}$ **b** $1-\frac{22}{25}$ **c** $1-\frac{9}{72}$ **d** $1-\frac{16}{36}$

3 Convert each fraction to a decimal.

a $\frac{17}{20}$ **b** $\frac{9}{36}$ **c** $\frac{25}{40}$ **d** $\frac{4}{15}$

4 Convert each fraction to a percentage.

a $\frac{10}{25}$ **b** $\frac{45}{72}$ **c** $\frac{4}{52}$ **d** $\frac{16}{48}$

5 How many:

a cards in a normal deck of playing cards?
b numbers on a die?
c months in a year?
d letters in the alphabet?
e Aces in a deck of playing cards?
f vowels in the alphabet?

6 A sock drawer contains 8 white socks, 2 black socks, 4 grey socks and 6 blue socks.

a What fraction of the socks are grey?
b What percentage of the socks are blue?
c What fraction of the socks are not white?
d What percentage of the socks are black or blue?

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7 A sample of students was surveyed on the number of cars owned by their families and the results are shown in the table.

Number of cars	Frequency
0	4
1	16
2	11
3	0
4	1

a Illustrate this data on a frequency histogram.

b What fraction of the sample owned no cars?

c What percentage of the sample owned three cars?

d What percentage (correct to one decimal place) of the sample owned at least one car?

e What was the most frequent number of cars?

8 Rate each of the following events as being very unlikely (VU), unlikely (U), likely (L) or almost certain (AC).

a It will rain in your area tomorrow.

b A car number plate contains numbers.

c A mother has triplets.

d There is a traffic jam in Sydney on Friday morning.

e You will live to be 100 years old.

f You arrive at school on time on Monday.

g There is a hailstorm in your area this week.

h You will send an email today.

i A double six appears when a pair of dice is rolled.

j An adult is married.

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Games of chance

4.01 Probability of simple events

Probability or **chance** can be measured on a 'likelihood scale' between 'impossible' and 'certain', or on a numerical scale between 0 and 1.

0		$\frac{1}{2}$		1
impossible	unlikely	even chance	likely	certain

The language of chance

Sample space

Probability terminology

An **outcome** is the result of an experiment or game. When a die is rolled, there are six possible outcomes: 1, 2, 3, 4, 5 or 6.

The **sample space** is the set of all possible outcomes, for example {1, 2, 3, 4, 5, 6} when a die is rolled.

An **event** is a group of one or more outcomes, for example the event of rolling an even number on a die is {2, 4, 6}.

EXAMPLE 1

List the sample space and count the number of outcomes for each situation.

a tossing a coin

b the colour shown on a traffic light

c the last digit of a phone number

Solution

a {head, tail}, 2 outcomes

b {red, amber, green}, 3 outcomes

c {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, 10 outcomes

Equally likely outcomes

We assume that all outcomes of an experiment or game are **equally likely** (have an equal chance). However, sometimes this is not the case because:

- a biased coin may show heads more often
- loaded dice show some numbers more often
- in a competition some players or teams may be more skilled.

Can you think of another situation where outcomes are not equally likely?

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EXAMPLE 2

A letter of the alphabet is chosen **at random** from the page of a magazine.

'At random' means that every letter on the page has an equal chance of being chosen.

a How many outcomes are there in the sample space?

b Are all the outcomes equally likely?

Solution

a There are 26 outcomes, because there are 26 letters in the alphabet.

b The outcomes are not equally likely, because some letters appear in words more often (for example, 'e').

Probability of an event

The probability of an event occurring, where all outcomes in the sample space are equally likely, is given by the formula:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

Probability can be expressed as a fraction, a decimal or a percentage.

EXAMPLE 3

A bag of jelly lollies contains 6 red lollies, 7 green lollies, 6 orange lollies, 5 yellow lollies and 4 white lollies. If one lolly is chosen at random, what is the probability that it is:

a red?

b orange or yellow?

c blue?

d not white?

Solution

Total number of outcomes = 6 + 7 + 6 + 5 + 4 = 28

There are 28 lollies and, therefore, 28 possible outcomes in the sample space.

a $P(\text{red}) = \frac{6}{28} = \frac{3}{14}$ 6 red lollies

b $P(\text{orange or yellow}) = \frac{6+5}{28} = \frac{11}{28}$

c $P(\text{blue}) = \frac{0}{28} = 0$ No blue lollies

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d $P(\text{not white}) = \dfrac{6+7+6+5}{28}$ 24 non-white lollies

$= \dfrac{24}{28}$

$= \dfrac{6}{7}$

Exercise 4.01 Probability of simple events

Example 1

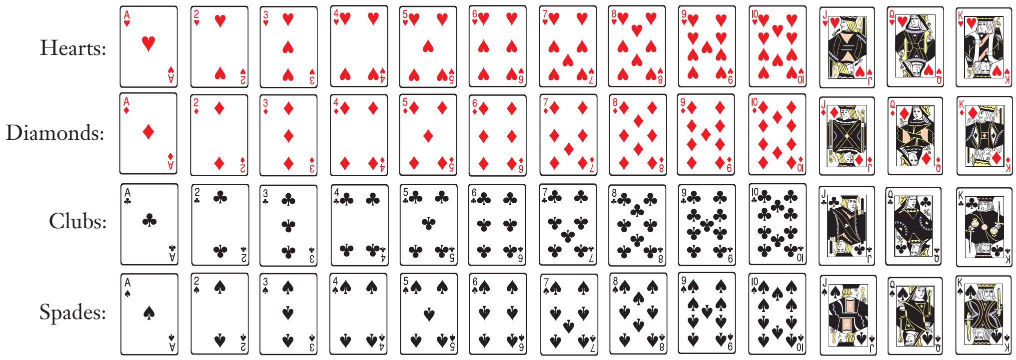
1 List the sample space and write the number of outcomes for each of the following situations.

- **a** selecting a day of the week to go shopping
- **b** choosing a letter from the word 'MATHEMATICS'
- **c** the classification rating of a movie
- **d** the gender of the next baby born at a hospital
- **e** the result of a driving test

2 Count the number of outcomes in the sample space for each of these.

- **a** the consonants from the alphabet
- **b** choosing the month to set a wedding date
- **c** choosing a four-digit PIN for accessing an ATM
- **d** the Year group of a primary school student
- **e** rolling a die in the shape of a triangular pyramid

3 A normal deck of playing cards contains 52 cards. There are four suits of 13 cards each.



One card is drawn at random from a deck. Count the number of possible outcomes for:

- **a** a red card
- **b** a 10
- **c** an even number
- **d** a black Ace
- **e** a picture card
- **f** the Queen of hearts

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Example 2

4 If England plays Brazil in a soccer match:

a what are the three possible outcomes in the sample space for the result of the game?

b are all three outcomes equally likely?

5 Explain what is wrong with the following statement: 'Since the weather is either fine or raining each day, the probability of a fine day is $\frac{1}{2}$.'

Example 3

6 A bag contains 6 blue marbles and 3 white marbles. If one marble is selected without looking, what is the probability that it is:

a blue? **b** white? **c** red? **d** blue or white?

7 The numbers 1 to 15 are written on separate cards. A card is selected at random. What is the probability that this number:

a is even? **b** is odd?

c is 9? **d** contains the digit 1?

8 A letter is chosen at random from the words 'NEW CENTURY'. Express as a decimal the probability that the chosen letter is:

a E **b** a vowel

c W or Y **d** a letter after N in the alphabet.

9 A multiple-choice question has answers labelled A, B, C and D. If you know the answer is not B, what is the probability of guessing the correct answer?

10 Adriana's purse has four \$2 coins, one \$1 coin, two 50-cent coins, six 20-cent coins, eight 10-cent coins and three 5-cent coins. If she takes out a coin at random, what is the percentage probability that it is:

a a 10-cent coin? **b** a \$1 coin? **c** a silver coin?

d not a 20-cent coin? **e** a \$2 coin or a 50-cent coin?

11 Ken, Lydia, Melanie, Nathan, Olivia and Paula wrote their names on separate cards. Which of the following is the probability that a card chosen at random will have a girl's name on it? Select **A**, **B**, **C** or **D**.

A 25% **B** $33\frac{1}{3}\%$ **C** 50% **D** $66\frac{2}{3}\%$

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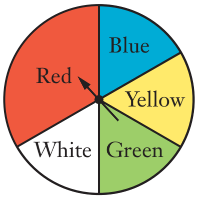
12 This spinner has a red sector that is twice as large as each sector of another colour. If the spinner is spun, find the probability that the arrow points to:

a red **b** white **c** red or green **d** not blue.



13 One card is selected at random from a deck of playing cards. What is the probability it is:

a a red card? **b** a black picture card?
c a King? **d** a number card?
e a red 4? **f** the Ace of spades?
g the Queen of hearts or the Jack of clubs? **h** a diamond?

14 A total of 1200 raffle tickets were sold at the school fete. Ramy bought eight tickets. What is the probability that he wins first prize?

15 What is the probability that a person chosen at random was born in a month beginning with A? Express the answer as a recurring decimal.

16 Ryan says that, because there are 16 teams in the NRL football competition, his favourite team, the Eels, has a chance of $\frac{1}{16}$ of winning the competition this year. Why is he incorrect in saying this?

17 A number from 1 to 20 is chosen at random by a computer. Find the percentage probability that it is:

a greater than 8 **b** a square number **c** divisible by 3.

18 A loaded die is biased so that it lands on 6 twice as often as it lands on any other number. Find the probability that, when it is rolled, it lands on:

a 6 **b** an even number **c** 5
d a number greater than 2 **e** a number other than 6 **f** 7

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Heads and tails

TECHNOLOGY

Tossing a coin

A spreadsheet can be used to simulate tossing a coin 20 times, by making the spreadsheet generate, at random, either the number 0 (for tails) or 1 (for heads).

Type in cell A1 of a new spreadsheet the bold heading 'Tossing a coin 20 times', and in cell A2, type '0 = tails, 1 = heads'.

In cell B4, enter the formula **=INT(RAND()*2)** to generate a random number 0 or 1 in that cell.

For an explanation of the INT and RAND functions on a spreadsheet, see 'Technology: Generating random numbers' on page 160.

Fill right to copy the formula across the row to cell F4 and simulate four more tosses of the coin.

Fill down to copy the formulas from rows 4 to 7, for a total of 20 tosses. This should give a random series of 0s and 1s, similar to the spreadsheet shown below.

	A	B	C	D	E	F
1	**Tossing a coin 20 times**					
2	0 = tails, 1 = heads					
3						
4		1	1	0	0	1
5		1	0	0	0	1
6		1	0	0	1	0
7		0	0	0	1	0

Decide how many times you would you expect heads (1) to come up in 20 tosses of a coin. Are there close to this many 1s in your 20 tosses?

To simulate the toss of the coin another 20 times, press the **F9** key to 'recalculate' another set of random numbers. Count how many 'heads' this time. (Is the number close to 10?)

Press the **F9** key each time you want to toss the coin another 20 times. Count how many heads each time.

We can make the spreadsheet count the number of heads by adding all 20 random values together, since heads = 1, tails = 0. In cell B9, type 'Number of heads' and in cell C9, type **=sum(B4:F7)**.

Press the **F9** key a few more times to see how many heads appear in every 20 tosses of the coin.

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Tree diagrams

4.02 Tables and tree diagrams

A **multi-stage experiment** has two or more stages occurring, for example:

- rolling 2 dice (2 stages)
- tossing a coin 3 times (3 stages)
- observing the weather for each day of a long weekend (3 stages)
- drawing out 5 winning raffle tickets from a barrel. (5 stages)

A **multi-stage event** consists of 2 or more events occurring together, such as rolling a sum of 10 on a pair of dice, or getting rain 3 days in a row. The sample space of a multi-stage experiment can be found using a **list**, a **table** or a **tree diagram**.

EXAMPLE 4

A coin is tossed twice.

a Show all the outcomes in the sample space using:

i a list **ii** a table **iii** a tree diagram.

b What is the probability of tossing:

i 2 heads? **ii** at least one tail?

Solution

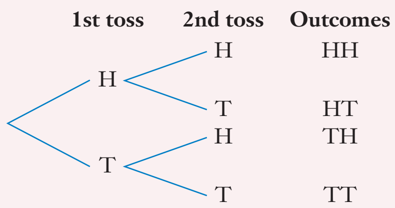
a Let H = heads, and T = tails.

i Using a list:
Sample space = {HH, HT, TH, TT}

ii Using a table:

		1st toss	
2nd toss		**H**	**T**
	H	HH	TH
	T	HT	TT

iii Using a tree diagram:



A tree diagram shows all the alternative pathways at each stage.
For the 1st toss, you could get heads or tails. If heads, then for the 2nd toss you could get heads or tails. If you get tails for the 1st toss, then for the 2nd toss you could get heads or tails.

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b **i** $P(\text{2 heads}) = P(\text{HH})$

$$= \frac{1}{4}$$

ii $P(\textbf{at least} \text{ one tail}) = P(\text{HT or TH or TT})$

$$= \frac{3}{4}$$

'At least one' means 1 or more (that is, not none).

Tables and tree diagrams are effective because they ensure that all possible arrangements are included.

A **table** can list the possible outcomes of a *two-stage* experiment.

A **tree diagram** lists the possible outcomes of an experiment with *two or more stages*. Branches stretch out to show the possible pathways of outcomes at each stage. An outcomes column at the end of the diagram lists the sample space.

EXAMPLE 5

Tables and tree diagrams

There are 8 people in a dance class: 3 men (Grant, Rick, Stefan) and 5 women (Bethany, Tania, Elise, Chloë, Felicity).

a List all possible male–female dancing couples, using:

i a table **ii** a tree diagram.

b How many dancing couples are possible?

c If one of these dancing couples is selected at random, what is the probability that:

i it is Rick and either Tania or Chloë?

ii it does not include Grant or Felicity?

iii it includes Stefan but not Bethany?

Solution

a Let: G = Grant R = Rick S = Stefan B = Bethany
T = Tania E = Elise C = Chloë F = Felicity

i Using a table:

		Men		
		G	**R**	**S**
Women	**B**	GB	RB	SB
	T	GT	RT	ST
	E	GE	RE	SE
	C	GC	RC	SC
	F	GF	RF	SF

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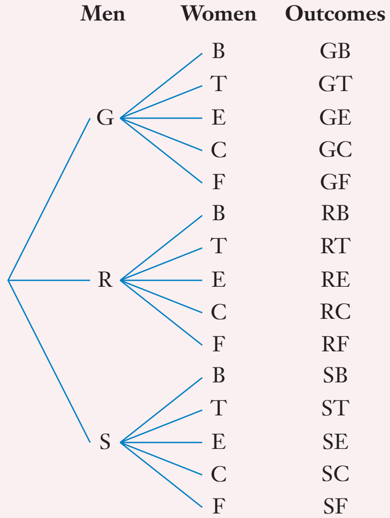
ii Using a tree diagram:



b There are 15 possible dancing couples.

Note that, when there are 3 men and 5 women, there are $3 \times 5 = 15$ possible couples.

c **i** $P(\text{Rick, Tania/Chloë}) = P(\text{RT or RC})$

$$= \frac{2}{15}$$

ii $P(\text{not Grant, not Felicity}) = P(\text{all outcomes without a G or F})$

$$= \frac{8}{15}$$

iii $P(\text{Stefan, not Bethany}) = P(\text{ST or SE or SC or SF})$

$$= \frac{4}{15}$$

Exercise 4.02 Tables and tree diagrams

1 Match each description (**a** to **i**) below with the correct set of *all* counting numbers that meet that description (**A** to **G**).

a at least 3	**b** from 1 to 4	**c** greater than 3	**d** 3 or more
e between 1 and 4	**f** at most 3	**g** 3 or less	**h** less than 3
i fewer than 5			
A {2, 3}	**B** {0, 1, 2, 3}	**C** {0, 1, 2, 3, 4}	**D** {3, 4, 5, …}
E {4, 5, 6, …}	**F** {0, 1, 2}	**G** {1, 2, 3, 4}	

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Example 4

2 **a** How many outcomes are possible when:

i tossing a coin? **ii** tossing a die?

b A coin and a die are tossed together. Show all outcomes in the sample space, using:

i a list **ii** a table **iii** a tree diagram.

c How many outcomes are in the sample space?

d Find the probability of tossing:

i heads and an even number

ii tails and a number less than 3

iii heads and not 1.

3 The digits 4, 5, 7 and 8 are written on separate cards and placed in a box. Two cards are drawn out at random to form a two-digit number. Note that the same digit cannot be used twice, so '75' is allowed but '77' is not.

a Use a tree diagram to list all possible two-digit numbers.

b How many two-digit numbers are possible?

c Find, as a percentage, the probability that the number drawn:

i has a first digit of 7 **ii** is odd

iii is less than 60 **iv** is divisible by 5.

4 A coin is tossed three times.

a Show all the possible outcomes of heads (H) and tails (T) using:

i a list

ii a tree diagram with three stages.

b How many outcomes are there in the sample space?

c Find, as a decimal, the probability of tossing:

i exactly one tail

ii at least two heads

iii three heads

iv more tails than heads.

5 How many possible outcomes are there when a coin is tossed:

a once? **b** two times?

c four times? **d** 10 times?

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Example 5

6 Two dice are rolled together and their sum is calculated.

a Copy and complete this table of possible sums.

		1st die					
2nd die	+	1	2	3	4	5	6
	1						
	2						
	3						
	4						
	5						
	6						

b What is the total number of possible outcomes?

c Find the probability of rolling a sum:

i of 10 **ii** of 6 **iii** less than 4

iv of 1 **v** that is odd **vi** of 7 or 11.

d Find the percentage probability of rolling:

i a 'double one' (1 and 1)

ii a 'double' anything (the same number twice).

e Which sum(s) is:

i most likely? **ii** least likely?

7 For breakfast, when staying at a hotel, Simone can choose one item from each course of a set menu.

1st course	2nd course
Cereal (C)	Bacon and eggs (B)
Raisin toast (R)	Pancakes (P)
Watermelon (W)	Sausages and hash browns (S)
Yoghurt (Y)	

a Use a table to list all the different two-course breakfasts available.

b How many two-course breakfasts are possible?

c If one of the combinations is chosen at random, find the probability that it includes:

i cereal or raisin toast

ii watermelon but not pancakes

iii yoghurt or pancakes

iv yoghurt and pancakes.

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8 Harry, Rina, Alison, Kirstie and Thuc are on the formal committee. They need to select a chairperson and a secretary.

a Use a table to determine all the possible pairings of chairperson and secretary. (Remember that a person cannot hold both positions.)

b If each person is equally likely to be selected, find the decimal probability that:

i Kirstie is chairperson and Thuc is secretary

ii Rina is either chairperson or secretary

iii Alison is secretary

iv Harry and Thuc fill the positions (in any order).

9 Consider a family with three children.

a Use a tree diagram to list the possible order of boys and girls for the children in the family.

b Which of the following is the number of possible outcomes? Select **A**, **B**, **C** or **D**.

A 8 **B** 16 **C** 7 **D** 6

c Find the probability of having:

i no boys

ii exactly two boys

iii at least one girl

iv at most one girl

v the eldest and youngest both being boys

vi more than one girl.

10 Suppose that the weather forecast for each day of the week is sunny, cloudy or raining, each being equally likely.

a Use a tree diagram to show all the possible outcomes for the weather for Saturday and Sunday, the two days of the weekend.

b Which of the following is the number of stages in this experiment? Select **A**, **B**, **C** or **D**.

A 2 **B** 3 **C** 4 **D** 6

c How many outcomes are possible?

d Find the probability that:

i it rains on both days

ii the weather is the same on both days

iii it doesn't rain on both days

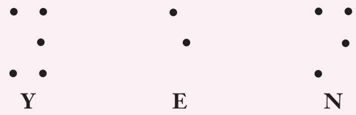
iv it is sunny on at least one of the days

v it is cloudy on one of the days and sunny on the other.

DID YOU KNOW?

The Braille alphabet

The Braille alphabet is a system of raised dots that allows blind people to read by touch. Letters and other symbols are represented by a cell of 1 to 6 dots printed in two columns. For example, the cells for the letters Y, E and N are:



There are also Chinese, Russian, Vietnamese and Korean versions of Braille. Frenchman Louis Braille invented the Braille alphabet in 1821. He became blind at 4 years of age after poking his eye accidentally with a saddler's awl (a tool for making holes in leather).

How many different cells are possible using the Braille system? (*Note*: A completely blank cell is allowed.)

4.03 Complementary events

The range of probability

Because the probability of an event is a fraction, as shown below, its value ranges from 0 to 1 (or, as a percentage, from 0% to 100%):

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

- A **certain** event must occur and has a probability of 1.
- An **impossible** event cannot occur and has a probability of 0.

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EXAMPLE 6

A die is rolled. Find:

a $P(\text{rolling a 3}) + P(\text{not rolling a 3})$

b $P(\text{rolling a number less than 5}) + P(\text{rolling 5 or more})$

c $P(\text{rolling an odd number}) + P(\text{rolling an even number})$.

Solution

a $P(3) + P(\text{not } 3) = P(3) + P(1, 2, 4, 5, 6) = \frac{1}{6} + \frac{5}{6} = 1$

b $P(< 5) + P(\geq 5) = P(1, 2, 3, 4) + P(5, 6) = \frac{2}{3} + \frac{1}{3} = 1$

c $P(\text{odd}) + P(\text{even}) = P(1, 3, 5) + P(2, 4, 6) = \frac{3}{6} + \frac{3}{6} = 1$

The probability of an event not occurring

'Rolling 3 on a die' and 'not rolling 3 on a die' are examples of **complementary events** (or one event and its 'opposite'). 'Complement' means something that completes or match with something to make a whole, and complementary events cover all the possibilities of a experiment. Because of this, the probabilities of complementary events must add to 1. Other examples of complementary events are:

- 'rolling a number less than 5', and 'rolling a number 5 or more'
- 'rolling an odd number', and 'rolling an even number'.

The 'complement of an event E' means all those outcomes that are not E, written as $\overline{E}$, and $P(\overline{E})$ is the probability of E *not occurring*.

Complementary events

$$P(\text{event does occur}) + P(\text{event does not occur}) = 1$$

$$P(\text{event does not occur}) = 1 - P(\text{event does occur})$$

$$P(\overline{E}) = 1 - P(E)$$

If the probability is in percentage form, then $P(\overline{E}) = 100\% - P(E)$.

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EXAMPLE 7

A ball is selected at random from a bag containing five green, seven red and two yellow balls.

Find the probability that the ball selected is:

a yellow **b** not yellow **c** blue

d not blue **e** not red **f** not red or not green.

Solution

Total number of balls = 5 + 7 + 2 = 14

a $P(\text{yellow}) = \frac{2}{14} = \frac{1}{7}$

b $P(\text{not yellow}) = 1 - P(\text{yellow})$

$= 1 - \frac{1}{7}$

$= \frac{6}{7}$

c $P(\text{blue}) \quad = \frac{0}{14} = 0$ There are no blue balls. This is an impossible event.

d $P(\text{not blue}) = 1 - P(\text{blue})$

$= 1 - 0$

$= 1$ All balls are not blue. This is a certain event.

e $P(\text{not red}) = 1 - P(\text{red})$

$= 1 - \frac{7}{14}$

$= \frac{7}{14}$

$= \frac{1}{2}$

f $P(\text{not red or green}) = 1 - P(\text{red or green})$

Why is this answer the same as for part **a**?

$= 1 - \frac{7+5}{14}$

$= \frac{1}{7}$

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Exercise 4.03 Complementary events

1 Write the complementary event for each of the following events.

a getting a tail when a coin is tossed
b selecting a heart card from a deck of playing cards
c failing a driving test
d having over three children in your family
e selecting a black sock from a drawer of black socks and white socks
f the weather today being rainy
g winning a swimming race
h a traffic light showing green

2 A die is rolled. What is the probability that the result is:

Example 6

a not a 6? **b** not less than 3? **c** not a multiple of 3?

3 Two coins are tossed together. List the sample space, then find the probability that the result is:

a two tails **b** not two tails **c** one tail
d not one tail **e** no tails **f** at least one tail.

4 Repeat Question **3** but with three coins tossed together.

5 A computer randomly selects a number from 1 to 20. Find the decimal probability that it is:

Example 7

a not less than 10 **b** not divisible by 3 **c** not a two-digit number
d not containing a 7 **e** not divisible by 5 **f** not a factor of 20.

6 Samantha bought a ticket in a raffle in which 1000 tickets were sold. If there are eight prizes, which of the following is the probability that Samantha doesn't win a prize? Select **A**, **B**, **C** or **D**.

A $\frac{124}{125}$ **B** $\frac{7}{8}$ **C** $\frac{1007}{1008}$ **D** $\frac{999}{1000}$

7 Each of the letters in the word 'PROBABILITY' is written on a piece of paper and put into a box. What is the probability that a letter drawn out at random is:

a not 'P'? **b** not 'B'?
c not a vowel? **d** not 'A' or 'B'?

8 What is the probability that a baby chosen at random from a maternity hospital was not born on a Saturday or Sunday?

9 The ratio of green to yellow to white lollies in a jar is 4 : 3: 2. One lolly is selected at random. Find the probability that it is:

a not green **b** yellow **c** not white
d not yellow **e** not red **f** not green or yellow.

10 In the dice game 'Craps', players lose if they roll a sum of 2 or 12 on a pair of dice. Which of the following is the probability of not losing? Select **A**, **B**, **C** or **D**.

A $\frac{5}{6}$ **B** $\frac{17}{18}$ **C** $\frac{35}{36}$ **D** $\frac{8}{9}$

11 The probability of rain this weekend is 23%. What is the probability that it will not rain?

12 A deck of cards is shuffled and one card is selected at random. Find:

a P(not a 10) **b** P(not a picture card) **c** P(not a club)

d P(not a black card) **e** P(not a red Ace) **f** P(not a black 7, 8, 9 or 10).

13 A traffic light is red for 63 seconds, green for 99 seconds and amber for 2 seconds. Express, correct to three decimal places, the probability that a car faces a traffic light that is:

a red

b not red

c not amber

d not green

e not red or amber

f not red, green or amber.

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14 What is the probability that a student chosen, at random, from your class was not born on 3 June?

15 What is the percentage probability that a mobile phone number selected at random does not end in 0 or 9?

16 In a football match, the Knights have a 42% chance of beating the Dragons, while the Dragons have a 51% chance of beating the Knights.

a What other outcome is possible?

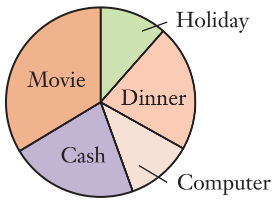
b What is the probability of this outcome?

17 The probability that Westvale High will win the debate is $\frac{13}{20}$. What is the probability that Westvale High will not win?

18 Lisa spins the wheel shown to determine her prize. What is the probability that she:

a does not win the holiday?

b does not win the cash or computer?



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4.04 Relative frequency

Experiments or surveys are used to predict the probability of an event when it is not feasible to calculate the probability of that event, for example, testing the effectiveness of batteries or predicting the outcome of a football match. The values obtained from repeated trials of an event are used to find the **experimental probability**. We call this the **relative frequency** of the event.

PS Matching probabilities

WS A page of spinners

WS Coins probability

WS Dice probability

WS Greedy Pig game

Relative frequency

$$\text{Relative frequency of an event} = \frac{\text{frequency of the event}}{\text{total frequency}}$$

EXAMPLE 8

This table shows the times of day that car accidents occurred in Uptown last month.

Time of accident	Frequency
midnight–4.00 a.m.	4
4.00 a.m.–8.00 a.m.	13
8.00 a.m.–noon	17
noon–4.00 p.m.	20
4.00 p.m.–8.00 p.m.	23
8.00 p.m.–midnight	7

Based on these results, what is the relative frequency of a car accident in Uptown occurring:

a between midnight and 4.00 a.m.?

b between 8.00 a.m. and 4.00 p.m.?

c after midday?

d earlier than 8.00 p.m.?

Solution

Total frequency = 4 + 13 + 17 + 20 + 23 + 7 = 84

There were 84 road accidents that month.

a $P(\text{midnight–4.00 a.m.}) = \frac{4}{84} = \frac{1}{21}$

b $P(\text{8.00 a.m.–4.00 p.m.}) = \frac{17+20}{84} = \frac{37}{84}$

c $P(\text{after midday}) = \frac{20+23+7}{84} = \frac{50}{84} = \frac{25}{42}$

d $P(\text{earlier than 8.00 p.m.}) = 1 - P(\text{after 8.00 p.m.})$

$$= 1 - \frac{7}{84}$$

$$= \frac{11}{12}$$

EXAMPLE 9

A pair of dice was rolled 50 times and their sum calculated each time. The results are shown in this table.

Sum	2	3	4	5	6	7	8	9	10	11	12
Frequency	0	2	4	6	5	5	9	6	8	3	2

a Draw a dot plot to illustrate this data.

b Which sum is:

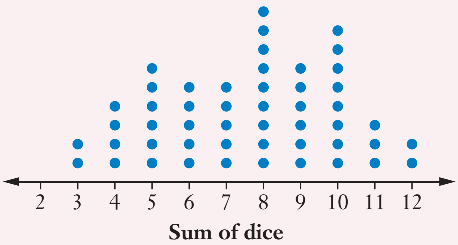
i most likely? **ii** least likely?

c Calculate, as a decimal, the relative frequency of rolling a sum:

i of 7 or 11 **ii** that is an odd number **iii** greater than 9.

Solution

a



b **i** 8 is most likely — It has the highest frequency (9).

ii 2 is least likely — It has the lowest frequency (0).

c **i** $P(7 \text{ or } 11) = \dfrac{5+3}{40} = \dfrac{8}{40} = 0.2$

ii $P(\text{odd}) = \dfrac{2+6+5+6+3}{40} = \dfrac{22}{40} = 0.55$

iii $P(>9) = \dfrac{8+3+2}{40} = \dfrac{13}{40} = 0.325$

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Exercise 4.04 Relative frequency

Example 8

1 A sample of matchboxes was taken and the number of matches in each box was counted. The results are shown in this table.

Number of matches	Frequency
48	2
49	15
50	70
51	56
52	6
> 52	1

iStock.com/Antagain

a How many matchboxes were in the sample?

b What is the relative frequency of a matchbox containing:

  i 51 matches?

  ii more than 50 matches?

  iii 49 or fewer matches?

2 A die was rolled 80 times, with the results shown below.

Outcome	1	2	3	4	5	6
Frequency	11	13	9	13	12	22

a Are the outcomes equally likely?

b Do you think this die is loaded? Give a reason for your answer.

c Express as a percentage the relative frequency of rolling 6 on this die.

d If this die was rolled 100 times, how many 6s would you expect?

3 Than surveyed the number of children in each house in her street, and found the following results:

Number of children	0	1	2	3	4	5
Frequency	5	10	14	7	3	1

a Which number of children is second-most likely?

b If a house is selected at random from Than's street, what is the relative frequency (as a decimal) that it contains:

  i two children? **ii** no children?

  iii at least one child? **iv** more than one child?

  v three or more children? **vi** at most three children?

c How many four-child families would you expect to find in a similar sample of 150 houses?

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4 In all of their home games, the Dragons won 16 matches and lost 12 matches. Which of the following is their experimental probability of losing a home game? Select **A**, **B**, **C** or **D**.

A 25% **B** $42\frac{6}{7}\%$ **C** $57\frac{1}{7}\%$ **D** 75%

5 The daily weather in Springfield during the month of June is shown in the table.

Weather	Number of days
Fine	13
Rain	10
Cloudy	7

a Are the outcomes equally likely?

b Find the relative frequency of the Springfield weather in June being:

i rainy

ii not cloudy.

6 At the 2016 Census, Australia's population was 23 401 892, including 11 855 248 females.

a What is the probability that a person chosen at random from the Australian population in 2016 was female? Express your answer as a percentage, correct to two decimal places.

b Is a person chosen from the 2016 Census more likely to be male or female? Why?

c When Australia's population reaches 25 000 000, how many *males* would you expect there to be, to the nearest thousand?

Example 9

7 Copy the following table and use it to record the results for throwing a drawing pin 30 times and observing whether it lands point up or point down.

Outcome	Tally	Frequency
Point up		
Point down		

a Are the outcomes equally likely?

b Graph the results on a sector graph.

c What is the relative frequency that a drawing pin lands point up?

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8 Copy and complete this table by noting the first letter of the surname of every student in your class.

Surname initial	Tally	Frequency
A–E		
F–J		
K–O		
P–T		
U–Z		

a Which class interval of initials is:

i least likely? **ii** most likely?

b What is the relative frequency of a student selected at random from the class having a surname beginning with a letter:

i from A to E? **ii** after O in the alphabet? **iii** not from K to O?

c If there are 120 students in Year 11, estimate how many students have a surname beginning with a letter from F to J.

9 Toss three coins together 40 times. Copy and complete this table by recording the number of tails that come up each time.

Alternatively, you could simulate tossing three coins together using a calculator or spreadsheet. (See 'Technology: Generating random numbers' next page.)

Number of tails	Tally	Frequency
0		
1		
2		
3		

a Are the outcomes equally likely? If not, which outcome is least likely?

b Express as a decimal the relative frequency of tossing:

i no tails **ii** fewer than three tails **iii** two heads.

c If three coins were tossed 100 times, estimate how many times two tails would result.

10 Draw a card at random from a deck of playing cards and record its suit in a table similar to the one shown. Do this 40 times, returning the card to the deck each time and reshuffling.

Suit	Tally	Frequency
Clubs		
Spades		
Hearts		
Diamonds		

a If a card is selected at random 40 times from a deck, how often should a hearts card be selected?

b Represent the data from your completed table on a sector graph.

c Find the relative frequency of selecting a card that is:

i a spade **ii** not hearts **iii** red.

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11 Copy and complete this table by recording the current ages of the students in your class.

Age	Tally	Frequency
15		
16		
17		
18		
19+		

a Represent this data on a dot plot.

b Which age is most frequent?

c Find the relative frequency of a student from your class:

i being aged 16 **ii** being aged below 18 **iii** not being aged 17.

d If there are 125 students in Year 11, estimate how many of them are 17 years old.

TECHNOLOGY

Generating random numbers

Random numbers can be generated on a calculator or spreadsheet using the RAN# or =RAND() functions. This function outputs a random decimal between 0 and 1 but we can use it to create a random number of any range.

To generate a random whole number from 0 to *n*

Multiply the random decimal generated (between 0 and 1) by (n + 1), then ignore the decimal part of the number (for example read 1.092 as 1).

For example, to create a random number from 0 to 4:

- on a scientific calculator, type RAN × 5 =. Press = repeatedly for more random numbers.
- on a graphics calculator, enter **int(ran#×5)** from the probability functions and press EXE. Press EXE repeatedly for more random numbers.

The INT function outputs the integer (whole) part of a decimal only.

- on a spreadsheet, type into a cell **=INT(RND()*5)** and press the Enter key. Note that, unlike other spreadsheet functions, =RAND() does not require a cell address inside its brackets.

To generate a random whole number from 1 to *n*

Multiply the random decimal generated (between 0 and 1) by n, then add 1, and ignore the decimal part of the number.

For example, to create a random number from 1 to 6:

- on a scientific calculator, type RAN × 6 + 1. Press = repeatedly for more random numbers.
- on a graphics calculator, enter int(ran#×6+1) from the probability functions and press EXE. Press EXE repeatedly for more random numbers.
- on a spreadsheet, type into a cell **=INT(RND()*6+1)** and press the Enter key.

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TECHNOLOGY

Rolling a pair of dice

A spreadsheet can be used to simulate the rolling of two dice and for calculating their sum.

Step 1: Enter the labels on the right into a blank spreadsheet.

Step 2: We will simulate the rolling of two dice 10 times so, in the 'Trial' column, we need the numbers 1 to 10. In cell A5, enter **=A4+1** to get 2, then use **Fill down** to copy that formula to cell A13 to generate the rest of the Trial numbers.

	A	B	C	D
1	**Rolling a pair of dice**			
2				
3	Trial	Die 1	Die 2	Sum
4	1			
5				
6				
7				
8				
9				
10				
11				

Step 3: In cell B4, enter **=INT(RAND()*6+1)** to generate a random integer from 1 to 6 and simulate the rolling of the first die. Copy or use **Fill right** to simulate the rolling of the second die and check that you have two numbers from 1 to 6 for each trial.

Step 4: In cell D4, enter an appropriate formula to calculate the sum for the pair of dice.

Step 5: **Fill down** the formulas in columns B, C and D to row 13 so that there are 10 trials for the rolling of two dice.

Step 6: To generate another 10 trials, press the **F9** key to 'recalculate' another set of random dice numbers. Do these numbers seem reasonable for the rolling of dice?

Step 7: Keep pressing the **F9** key to simulate rolling the die another 10 times and observe the sums are generated. Does 7 come up more often than other numbers?

Step 8: Extend this spreadsheet to simulate 100 trials of rolling a pair of dice.

Step 9: Modify this spreadsheet to simulate the tossing of two coins (tails = 0, heads = 1) and change the 'Sum' column so it calculates the 'Number of heads'.

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Rolling a die

Probability of $\frac{1}{2}$

4.05 Comparing relative frequency and theoretical probability

How is the relative frequency (experimental probability) of an event related to its theoretical probability?

EXAMPLE 10

A die is rolled 60 times and the results are as shown.

Outcome	1	2	3	4	5	6
Frequency	6	13	12	8	9	12

a Find the relative frequency (experimental probability) of rolling a 4:

- **i** as a fraction
- **ii** as a decimal, correct to three decimal places.

b Find the theoretical probability of rolling a 4:

- **i** as a fraction
- **ii** as a decimal, correct to three decimal places.

c Are the experimental and theoretical probabilities similar?

d If a die is rolled 60 times, what is the expected number of times a 4 will be rolled?

e The die is rolled 300 times, with the following results.

Outcome	1	2	3	4	5	6
Frequency	46	48	46	52	51	57

Calculate, to three decimal places, the relative frequency of rolling a 4.

f What do you notice about the relative frequency and theoretical probability now?

g If a die is rolled 300 times, what is the expected number of times a 4 will be rolled?

Solution

a Relative frequency:

i $P(4) = \frac{8}{60} = \frac{2}{15}$ **ii** $P(4) = 0.1333\ldots \approx 0.133$

b Theoretical probability:

i $P(4) = \frac{1}{6}$ **ii** $P(4) = 0.1666\ldots \approx 0.167$

c By examining the decimal values, the relative frequency and theoretical probability are close.

d Expected number of 4s $= \frac{1}{6} \times 60 = 10$

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e Experimental $P(4) = \frac{52}{300} = 0.1733... \approx 0.173$

f The relative frequency is now closer to the theoretical probability (0.167).

g Expected number of 4s $= \frac{1}{6} \times 300 = 50$

Relative frequency and theoretical probability

As the number of trials increases, the relative frequency becomes closer to the theoretical probability. This is sometimes called 'the law of averages'.

Exercise 4.05 Comparing relative frequency and theoretical probability

This exercise may be completed as a group activity.

1 Ethan tossed three coins together.

Example 10

a List all the possible outcomes in the sample space.

b Write, as a percentage, the probability of tossing:

i two heads **ii** one or three heads.

c If Ethan tossed three coins 88 times, how many times should he get:

i two heads? **ii** one or three heads?

d Ethan actually had the following results:

Number of heads	0	1	2	3
Frequency	11	35	30	12

Write, as a percentage correct to two decimal places, the relative frequency of getting:

i two heads **ii** one or three heads.

e How do the relative frequencies compare with the theoretical probabilities?

f If three coins are tossed together 200 times, what is the expected number of times that:

i two heads will be tossed? **ii** one or three heads will be tossed?

g Stephanie tossed three coins 200 times with the following results:

Number of heads	0	1	2	3
Frequency	23	72	80	25

Write, as a percentage, the relative frequency of tossing:

i two heads **ii** one or three heads.

h Do the relative frequencies become closer to the theoretical probabilities with more repeated trials?

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2 Roll a die 150 times and record your results in a table similar to the one below.

Alternatively, you could simulate rolling a die using a calculator or spreadsheet. (See 'Technology: Generating random numbers' on page 160.)

Outcome	Tally	Frequency
1		
2		
3		
4		
5		
6		

a Express as a decimal, correct to three decimal places:

i the relative frequency of rolling a 1

ii the theoretical probability of rolling a 1

iii the relative frequency of rolling an even number

iv the theoretical probability of rolling an even number.

b What is the expected number of times a 1 should result when a die is rolled 150 times?

c By pooling your results with those of other students or groups in your class, repeat part a to see whether, with more trials, your relative frequencies become closer to the theoretical probabilities.

3 **a** Copy and complete this table of the possible sums when a pair of dice are rolled together.

b As a decimal, correct to four decimal places, what is the theoretical probability of rolling a sum of:

i 4?

ii 2 or 12?

iii 7 or 11?

		1st die					
2nd die	+	1	2	3	4	5	6
	1						
	2						
	3						
	4						
	5						
	6						

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c Copy this table, then roll a pair of dice 120 times and record your results in the table.

Sum	Tally	Frequency
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

d If a pair of dice is rolled 120 times, what is the expected number of:

i 4s?

ii 2s or 12s?

iii 7s or 11s?

e As a decimal, correct to four decimal places, what is the relative frequency of rolling a sum of:

i 4?

ii 2 or 12?

iii 7 or 11?

4 A card is randomly drawn from a deck of cards to see if it is a picture card (Jack, Queen or King). This is done 78 times, with each card being replaced in the deck before the next draw. The results are shown in the table.

a Express as a decimal, correct to three decimal places:

i the calculated probability of selecting a picture card

ii the experimental probability (relative frequency) of selecting a picture card.

Outcome	Frequency
Picture card	20
Not a picture card	58

b If a card is randomly drawn from a deck of cards 78 times, which of the following is the expected number of times a picture card should be drawn? Select **A**, **B**, **C** or **D**.

A 18 **B** 21 **C** 24 **D** 26

c Use a deck of cards to perform this experiment 78 times, and check whether your experimental probability is close to the calculated probability.

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5 **a** How many outcomes are there in the sample space when a coin and a die are tossed together?

b Tahlia tossed a coin and a die together 48 times and found the frequencies shown.

		Die					
		1	**2**	**3**	**4**	**5**	**6**
Coin	**Heads**	4	5	3	2	4	5
	Tails	3	6	5	4	4	3

Calculate as a decimal, correct to three decimal places:

i the calculated probability of tossing a tail and a 3

ii the relative frequency of tossing a tail and a 3

iii the calculated probability of tossing a head and an even number

iv the relative frequency of tossing a head and an even number

v the calculated probability of tossing a tail and any number but 4

vi the experimental probability of tossing a tail and any number but 4.

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c If a coin and a die are tossed together 48 times, how many times should each of the following events occur?

i a tail and a 3

ii a head and an even number

iii a tail and any number but 4.

6 A coin is tossed repeatedly, and it lands with heads uppermost eight times in a row. On the next toss, is a head more likely to result again, or a tail? Or does each outcome still have an equal chance?

7 Zeinab uses a computer program to simulate tossing a coin but she suspects that the simulated coin is biased when she notices the pattern of results recorded in the table below.

Total number of tosses	50	140	200	540
Number of heads	23	55	84	226

a Calculate as a percentage, correct to one decimal place, the relative frequency of the computer simulation ‘tossing heads’ when the computer tosses the coin:

i 50 times **ii** 140 times **iii** 200 times **iv** 540 times.

b Is Zeinab justified in believing that the computer program’s coin is biased? Give a reason for your answer.

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INVESTIGATION

ARE THE LOTTO NUMBERS EQUALLY LIKELY?

Lotto is a gambling game run by NSW Lotteries, in which 45 numbered balls are placed in a barrel and eight numbers are selected at random from the barrel: six main numbers plus two supplementary numbers. Between the launch of the 45 number game and October 2016, there were a total of 9438 Lotto numbers drawn for the Saturday game. The frequency of drawing each number (from 1 to 45) over this period was as follows:

Number	Frequency	Number	Frequency	Number	Frequency
1	235	16	212	31	215
2	190	17	187	32	211
3	221	18	228	33	221
4	201	19	221	34	200
5	226	20	205	35	184
6	214	21	213	36	221
7	224	22	221	37	211
8	221	23	190	38	207
9	205	24	216	39	196
10	197	25	227	40	227
11	222	26	218	41	225
12	227	27	204	42	222
13	208	28	192	43	205
14	179	29	207	44	181
15	211	30	187	45	203

Source: www.nswlotteries.com.au

1 If, in this period, 9438 numbers were drawn, and each number from 1 to 45 was equally likely, how often would you expect each number to have been drawn?

2 How many of the frequencies from the above table are equal to the expected number you suggested in Question **1**?

3 Which Lotto number came up:

a most often? **b** least often?

4 Do you think the Lotto numbers are equally likely, or is the Lotto draw biased? Give a reason for your answer.

4.06 Probability tree diagrams

WS Tree diagrams

A **probability tree diagram** has the probabilities of each stage shown on the branches.

EXAMPLE 11

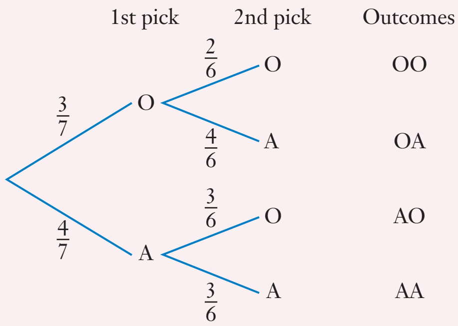
A fruit bowl contains three oranges and four apples. Prue selects two pieces of fruit at random from the bowl. Use a tree diagram to determine the probability that Prue selects:

a two oranges **b** an orange and an apple.

Solution

3 oranges, 4 apples: total 7

3O
4A



See explanation below for this probability tree diagram.

Suppose Prue picks an orange first.

$P(\text{1st pick is an orange}) = \frac{3}{7}$ — 3 oranges from 7 fruit

- $P(\text{2nd pick is orange}) = \frac{2}{6}$ — 2 oranges from 6 fruit left
- $P(\text{2nd pick is apple}) = \frac{4}{6}$ — 4 apples from 6 fruit left

Suppose Prue picks an apple first. — 4 apples from 7 fruit

$P(\text{1st pick is an apple}) = \frac{4}{7}$

- $P(\text{2nd pick is orange}) = \frac{3}{6}$ — 3 oranges from 6 fruit left

  $P(\text{2nd pick is apple}) = \frac{3}{6}$ — 3 apples from 6 fruit left

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a $P(\text{OO}) = \frac{3}{7} \times \frac{2}{6}$
$= \frac{1}{7}$

Multiply probabilities on OO branches

b $P(\text{OA or AO}) = P(\text{OA}) + P(\text{AO})$
$= \frac{3}{7} \times \frac{4}{6} + \frac{4}{7} \times \frac{3}{6}$
$= \frac{4}{7}$

Two ways of selecting an orange and apple

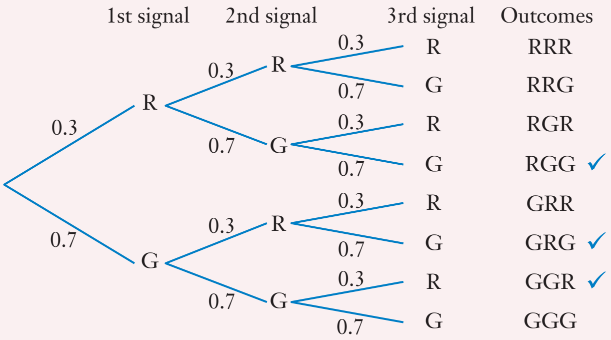
Note: Probabilities on branches from the same point always add to 1, for example, $\frac{3}{7} + \frac{4}{7}, \frac{2}{6} + \frac{4}{6}$ on the tree diagram above.

EXAMPLE 12

To drive to work, Ms Gough passes through three sets of traffic lights. The probability of a red signal (including amber) on each light is 0.3. Find the probability that on her way to work Ms Gough gets:

a all green lights **b** two green lights **c** at least one green light.

Solution



a $P(\text{GGG}) = 0.7 \times 0.7 \times 0.7$

$= 0.343$

b $P(\text{2 green}) = P(\text{RGG}) + P(\text{GRG}) + P(\text{GGR})$ — Ticked on the tree diagram

$= (0.3 \times 0.7 \times 0.7) + (0.7 \times 0.3 \times 0.7)$
$+ (0.7 \times 0.7 \times 0.3)$

$= 3 \times (0.3 \times 0.7 \times 0.7)$

$= 0.441$

c $P(\text{at least 1 green}) = 1 - P(\text{no green})$ — Using the complementary event rule

$= 1 - P(\text{RRR})$

$= 1 - (0.3 \times 0.3 \times 0.3)$

$= 0.973$

Probability tree diagrams

In a probability tree diagram:

- branches from the same point have probabilities that add to 1
- to calculate the probability of an outcome, multiply the probabilities along that branch
- to calculate the probability of two or more outcomes, add their calculated probabilities
- $P(\text{at least one}) = 1 - P(\text{none})$.

Exercise 4.06 Probability tree diagrams

Example 11

1 A jar contains five red lollies and three green lollies. Maya selects two lollies at random. Calculate the probability that they are of different colours. Select **A**, **B**, **C** or **D**.

A $\frac{13}{28}$ **B** $\frac{15}{32}$ **C** $\frac{15}{28}$ **D** $\frac{15}{56}$

2 Nikolai buys two tickets in a 50-ticket raffle with two prizes. What is the probability that he wins:

a 1st prize? **b** 1st and 2nd prizes?

c no prize? **d** at least one prize?

3 There are ten batteries in a box and two are flat. Danielle takes two batteries out of the box at random. Calculate the probability that:

a both batteries are flat **b** only one of the batteries is flat.

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4 A committee of four women and three men need to select a chairperson and a secretary. If every person is equally likely to be chosen, what is the probability that:

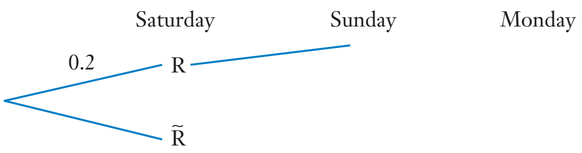
a both positions are filled by women?

b both positions are filled by men?

c the chairperson is female and the secretary is male?

5 For a long weekend (Saturday to Monday), the probability of rain on any day is 0.2. Example 12

a Copy and complete this tree diagram.



b Calculate the percentage probability that over the long weekend there is:

i exactly one rainy day **ii** no rainy days **iii** at least one rainy day.

6 A die is rolled three times. What is the probability that 6 does not come up in any roll?

7 A biased coin comes up tails 63% of the time. If the coin is tossed three times, calculate the probability, correct to three decimal places, that:

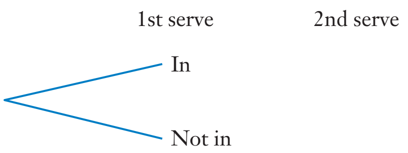
a a tail comes up every time **b** a tail comes up twice.

8 Five cards are numbered 1, 2, 3, 4 and 5. If two cards are randomly selected, what is the probability of selecting:

a two even numbers? **b** two odd numbers?

9 A tennis player gets a second serve only if her first serve is not in. Daria's first serve has a 0.78 probability of going in and her second serve has a 0.94 probability of going in.

a Copy and complete this tree diagram.



b A double fault occurs when both serves do not go in. What is the probability that Daria serves a double fault?

c What is the probability that one of Daria's serves goes in?

10 A student council has eight Year 10 students, six Year 11 students and four Year 12 students. Two students are selected from the council at random to represent the school at the mayor's lunch. Calculate the probability that:

a both students are from Year 10

b there is one student from each of Years 10 and 11

c at least one of the students is from Year 12

d each student is from a different Year.

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11 Three people are selected at random. What is the probability that all were born in March? Select **A**, **B**, **C** or **D**.

A 0.0579% **B** 1.56% **C** 2.78% **D** 3.70%

12 In a town, 8% of people have a virus that can be detected by a medical test which gives a correct reading 90% of the time. If Sam is tested for the virus, what is the percentage probability that he:

a has the virus but it is not detected by the test?

b has the virus and it is detected?

c does not have the virus but it is falsely detected?

INVESTIGATION

MORTALITY RATES

Insurance companies use **mortality rates** to calculate premiums for life **insurance** policies. Mortality rates are the probabilities that a person of a certain age will die that year. This table shows the mortality rates of Australians aged 0 to 100 in 2013–2015.

Age	Males	Females	Age	Males	Females
0	0.00354	0.00249	40	0.00093	0.00076
1	0.00024	0.00021	45	0.00135	0.00116
2	0.00018	0.00018	50	0.00232	0.00179
3	0.00015	0.00015	55	0.00388	0.00258
4	0.00013	0.00012	60	0.00616	0.00371
5	0.00011	0.00011	65	0.01010	0.00559
10	0.00006	0.00006	70	0.01688	0.00895
15	0.00015	0.00012	75	0.02819	0.01581
17	0.00030	0.00014	80	0.04729	0.02892
20	0.00048	0.00015	85	0.08123	0.05925
25	0.00051	0.00016	90	0.15332	0.11670
30	0.00056	0.00026	95	0.24057	0.20842
35	0.00070	0.00036	100	0.32793	0.29473

Source: Australian Bureau of Statistics. Life tables 2013–2015

a How do you think these figures were calculated?

b What patterns do you notice in the mortality rates as people get older? Why?

c Why do you think babies have a higher mortality rate?

d Why do you think there is a difference between male and female mortality rates?

e These mortality rates were lower than those in the year 2010, and this year's mortality rates will be lower than these ones. Why do you think this is so?

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SAMPLE HSC PROBLEM

Two dice are rolled together and the difference between the larger and smaller number is calculated.

a Copy and complete this table of possible differences.

		1st die					
		1	**2**	**3**	**4**	**5**	**6**
2nd die	**1**						
	2				2		
	3						
	4						
	5	4					
	6						0

b Which difference is most likely?

c Find the probability of rolling a difference:

i of 2

ii that is an odd number

iii less than 5.

Study tip

When and where to study

Are you an early bird or a night owl? do you work best in the morning, in the afternoon or at night? Identify your **peak performance period** and study during this time each day, so that it becomes a habit. There will be times when you can't study due to other commitments but it's important to develop a **regular routine** so that your family and friends know when you should not be interrupted.

Students often ask, 'How many hours should I study each night?', but it is the **quality** of study that is more important than the **quantity**. It is a question of the effort and commitment required, not the number of hours. The amount of time you should study is the amount of time necessary for you to fulfil all of your study tasks and demands. Some students like to do the same amount each day, some do more or less on weekends, or have one day that is study-free. Find out what works best for you, and stick to it.

Students often study by themselves in their rooms or at the library. Some prefer company and use the dining table, because they like the noise and space. Others like to sit outside in the fresh air. You can choose different places for different types of homework.

A good study place has:

- plenty of space to spread out work, such as on a big desk
- minimal noise, few distractions and interruptions
- good lighting and ventilation, and is neither too hot nor too cold
- comfortable and supportive seating.

Study is a serious business. You will be concentrating for a while, so choose a place where you won't be easily distracted. Use a good desk lamp and open the window to prevent sore eyes and drowsiness.

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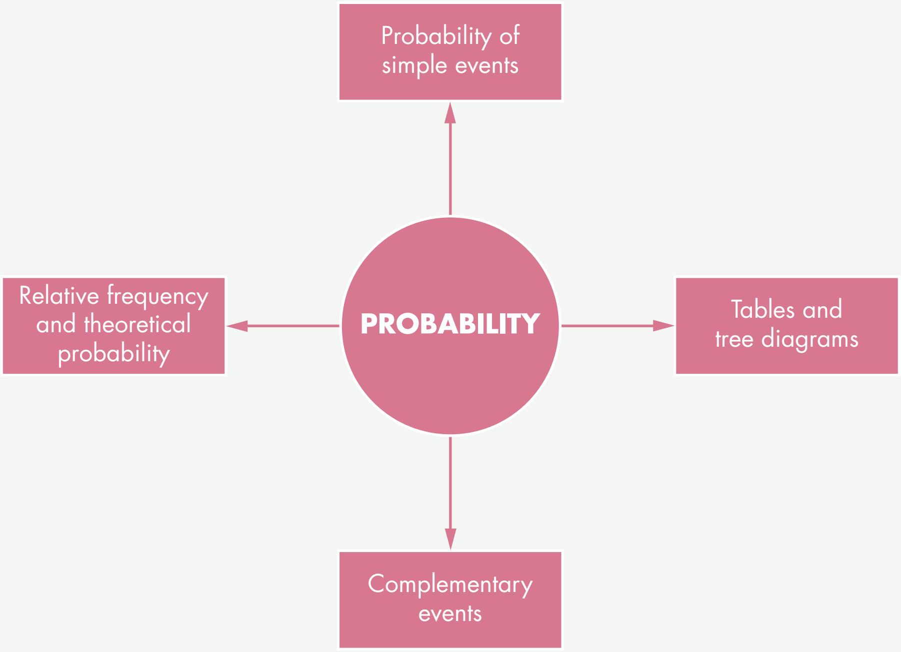
4. CHAPTER SUMMARY

This chapter, Probability, looked at the probabilities of simple and multi-stage events. You should have a good understanding of probability concepts, including complementary events and determining sample spaces using lists, tables and tree diagrams. You should be able to find and compare theoretical probabilities and relative frequencies using given data or through repeated trials of an experiment.

Probability crossword

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map is shown below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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WS Probability review

4. TEST YOURSELF

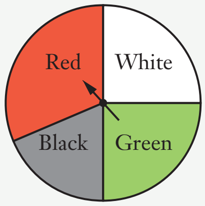
Exercise 4.01

1 A letter is selected at random from the alphabet. Find the probability that it is:

a P **b** a vowel or Y **c** not W, X, Y or Z.

Exercise 4.02

2 A spinner is divided into sectors as shown, with 'Red' taking up one-third of the circle. What is the percentage probability that the arrow points to:



a green?

b black?

c red or green?

d a sector that is not red?

Exercise 4.01

3 Explain what is wrong with the following statement: 'Since a person can be either married or single, the probability of a person being married is 50%'.

Exercise 4.02

4 **a** Use a tree diagram to list the sample space of the different possible combinations of boys (B) and girls (G) in a family of three children.

b If a three-child family is selected at random, what is the probability that it contains:

i all girls? **ii** exactly one boy?

iii more boys than girls? **iv** at least one girl?

v at most one girl? **vi** a boy as the middle child?

Exercise 4.02

5 Two dice are rolled together. What is the probability that the two dice:

a show the same number? **b** show a sum of 8?

Exercise 4.03

6 If Santi has a 68% chance of missing a basketball shot, what is the probability that she scores with the shot?

Exercise 4.03

7 In a batch of 240 instant (scratch) lottery tickets, 54 contain a prize. Which of the following is the decimal probability of not winning a prize from one of these tickets? Select **A**, **B**, **C** or **D**.

A 0.225 **B** 0.443 **C** 0.46 **D** 0.775

Exercise 4.03

8 A sample of students was surveyed about how they travelled to school. The results are shown in the table.

Outcome	Frequency
Walk	43
Car	30
Bicycle	18
Bus	28
Train	21

What is the probability that a student, chosen at random, travels by:

a car?

b public transport?

c a method other than bicycle?

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9 A factory tests a batch of batteries and finds that 77 of them are good while 3 are faulty.

Exercise 4.04

a Express, as a percentage, the probability that a battery selected at random is good.

b If the factory makes 5000 batteries, how many would you expect to be faulty?

10 Roll a die 48 times and record in a table, similar to the one shown, the number of times the die lands with '6' uppermost.

Exercise 4.04

Exercise 4.05

Outcome	Tally	Frequency
6		
Not 6		

a Represent this data on a divided bar graph.

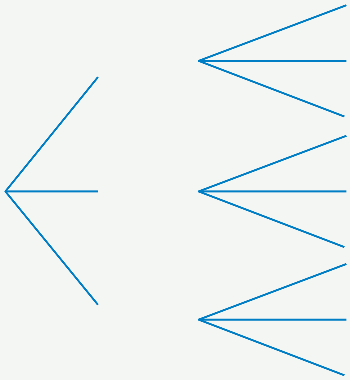
b Find the relative frequency of rolling a 6 on this die.

c What is the theoretical probability of rolling a 6 on a die?

d What is the expected number of 6s when a die is rolled 48 times?

11 Which situation can be illustrated by this tree diagram? Select **A**, **B** or **C**.

Exercise 4.06



A tossing a coin three times

B selecting two balls from a bag of red, blue and green balls

C three students passing or failing an exam.

12 Tegan buys five tickets in a raffle in which 80 tickets are sold. There are three prizes. Use a tree diagram to find the probability that Tegan:

Exercise 4.06

a wins all three prizes

b wins at least one prize

c wins exactly one prize.

Express answers as percentages, correct to two significant figures.

Qz

Chapter quiz

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