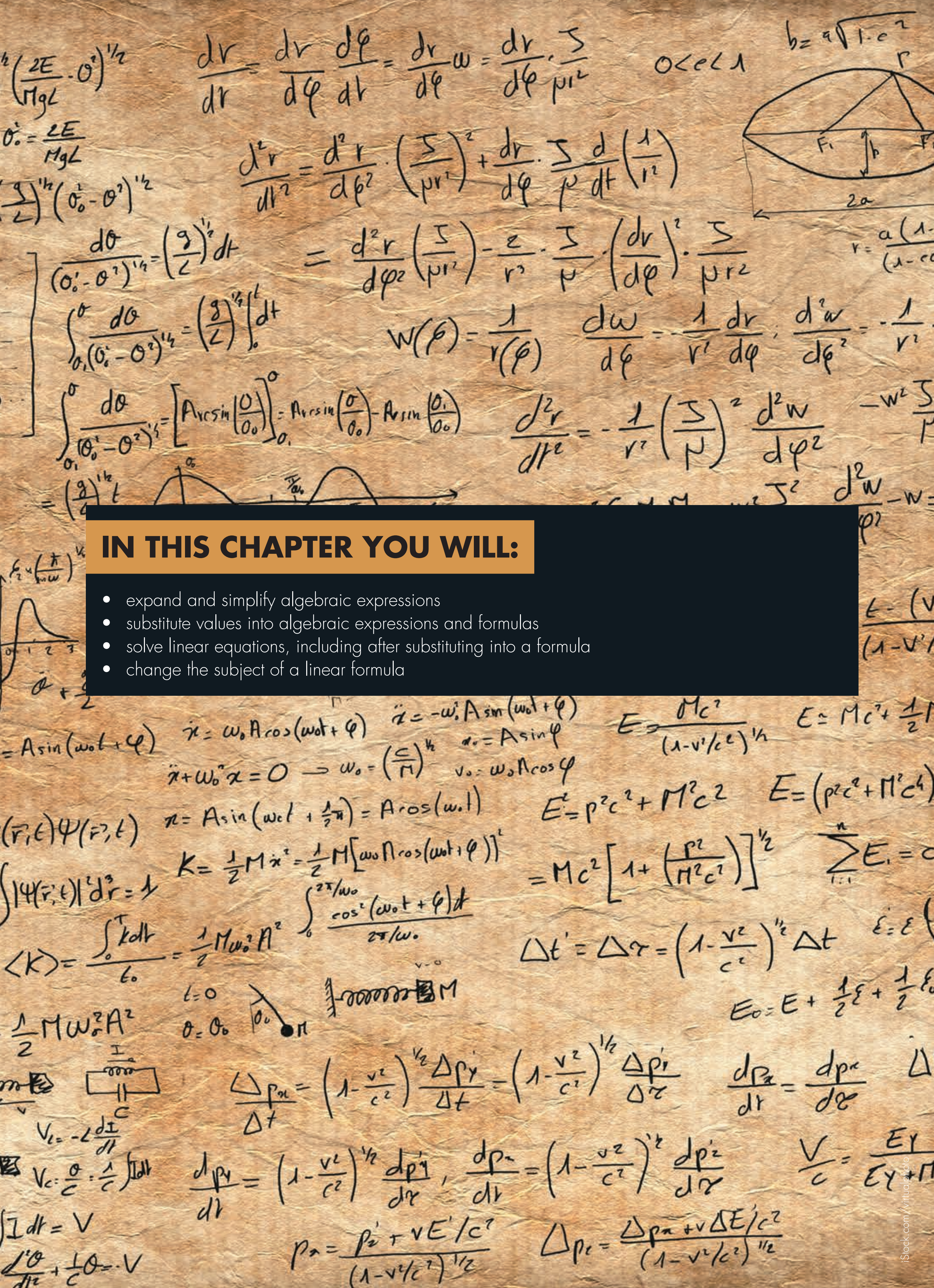
ALGEBRA

2.

FORMULAS AND EQUATIONS

German scientist Albert Einstein was only 26 years old when he proposed a new theory of physics for small particles of matter (such as atoms) moving at very high speeds. In 1905, he proposed that matter (mass) could be converted to large amounts of energy, describing this relationship with the formula $E = mc^2$, where E stands for energy, m stands for mass and c stands for the speed of light. Einstein's Theory of Special Relativity revolutionised conventional laws of physics and led to the development of nuclear energy.

CHAPTER OUTLINE



IN THIS CHAPTER YOU WILL:

- expand and simplify algebraic expressions
- substitute values into algebraic expressions and formulas
- solve linear equations, including after substituting into a formula
- change the subject of a linear formula

TERMINOLOGY

base
formula
power
simplify
variable
equation
LHS
RHS
subject
expand
like terms
rule
substitute

SkillCheck

Assignment 2

1 Evaluate each expression.

a $9 \times (-3)$ **b** $9 + (-3)$ **c** $2 - 10$

d $(-5)^2$ **e** $-24 \div 6$ **f** $-2 - (-4)$

2 If $d = 3$, evaluate each expression.

a $8d - 9$ **b** $\dfrac{d+6}{3}$ **c** $7 + 3d$

d $\sqrt{12d}$ **e** d^5 **f** $2d^2$

3 If $u = -2$, evaluate each expression.

a $u^2 - 4$ **b** $3(u + 1)$ **c** $8 - u$

d u^7 **e** $\dfrac{-18}{u}$ **f** $5 - 2u$

4 Simplify each expression.

a $4x + 7 - 3x - 10$ **b** $18 - 2a + a + 2$ **c** $5r + 12 - 15 - 5r$

5 Expand each expression.

a $3(2x - 1)$ **b** $7(4a + 5)$ **c** $-2(6p - 2)$

6 Solve each equation.

a $3b = -15$ **b** $w - 8 = 9$ **c** $2r + 4 = 22$

7 Evaluate $A = \frac{1}{2}(a + b)h$ if $a = 10$, $b = 15$ and $h = 3$.

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2.01 Simplifying algebraic expressions

An **algebraic expression** is made up of **terms** involving **variables** and numerals.
For example, $3x^2 - x + 10$ has three terms: $3x^2$, $-x$ and 10.

Collecting like terms

Algebra using diagrams

Working with like terms

- **Like terms** have exactly the same variables.
- Only like terms can be added or subtracted.

EXAMPLE 1

Simplify each expression.

a $2a^2 - a + 5 + 8a$ **b** $4kr - 6pr - pr + 10kr$ **c** $2xy + 4 - y + 4yx$

Solution

a $2a^2 - a + 5 + 8a = 2a^2 - a + 8a + 5$ — $-a$ and $8a$ are like terms

$= 2a^2 + 7a + 5$ — $2a^2$ and $-a$ are not like terms

b $4kr - 6pr - pr + 10kr = 4kr + 10kr - 6pr - pr$ — Grouping pairs of like terms

$= 14kr - 7pr$

c $2xy + 4 - y + 4yx = 2xy + 4yx - y + 4$ — $2xy$ and $4yx$ are like terms

$= 6xy - y + 4$

Multiplying terms

- When multiplying terms, multiply the numbers and variables separately.
- When multiplying terms with **powers** that have the same **base**, add the powers.

EXAMPLE 2

Simplify each expression.

a $-3bc \times 8ab$ **b** $4p^2q \times \dfrac{3p^3}{2}$ **c** $(-5r)^2$

Solution

a $-3bc \times 8ab = (-3 \times 8) \times (a \times b \times b \times c)$

$= -24ab^2c$

To **simplify**, first multiply the numbers. Then multiply the variables in alphabetical order.

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b $4p^2q \times \frac{3p^3}{2} = \frac{4 \times 3}{2} \times p^2 \times p^3 \times q$

$= 6p^5q$

$p^2 \times p^3 = p^5$
Add the powers.

c $(-5r)^2 = (-5r) \times (-5r)$

$= -5 \times (-5) \times r \times r$

$= 25r^2$

Dividing terms

- When dividing terms, divide the numbers and variables separately.
- When dividing terms with powers that have the same base, subtract the powers.

EXAMPLE 3

Simplify each expression.

a $\frac{8m^3n}{2m^2}$ **b** $-15w \div 3w^2$ **c** $\frac{6ak^3}{20ak}$

Solution

a $\frac{8m^3n}{2m^2} = \frac{8}{2} \times \frac{m^3n}{m^2}$

$= 4mn$

$\frac{m^3}{m^2} = m^1 = m.$
Subtract the powers.

b $-15w \div 3w^2 = \frac{-15w}{3w^2}$

$= \frac{-15}{3} \times \frac{w}{w^2}$

$\frac{w}{w^2} = \frac{\cancel{w}^{1}}{w\cancel{w}} = \frac{1}{w}$

$= -5 \times \frac{1}{w}$

$= \frac{-5}{w}$

c $\frac{6ak^3}{20ak} = \frac{6}{20} \times \frac{ak^3}{ak}$

$= \frac{3}{10} \times k^2$

$= \frac{3k^2}{10}$

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EXAMPLE 4

Simplify each expression.

a $\dfrac{4a}{5} \times \dfrac{5h}{12a}$

b $\dfrac{3m}{10n} \div \dfrac{6m}{25n}$

Solution

a $\dfrac{4a}{5} \times \dfrac{5h}{12a} = \dfrac{\overset{1}{\cancel{4}}\,\cancel{a}}{\cancel{5}} \times \dfrac{\cancel{5}\,h}{\underset{3}{\cancel{12}}\,\cancel{a}}$

$= \dfrac{1 \times h}{3}$

$= \dfrac{h}{3}$

Each fraction cannot be simplified on its own, so simplify the numerator of each fraction with the **denominator** of the other fraction.

$\dfrac{4}{12} = \dfrac{1}{3}$, $\dfrac{a}{a} = 1$, $\dfrac{5}{5} = 1$

b $\dfrac{3m}{10n} \div \dfrac{6m}{25n} = \dfrac{3m}{10n} \times \dfrac{25n}{6m}$

When dividing by a fraction, multiply by its reciprocal.

$= \dfrac{\overset{1}{\cancel{3}}\,\cancel{m}}{\underset{2}{\cancel{10}}\,\cancel{n}} \times \dfrac{\overset{5}{\cancel{25}}\,\cancel{n}}{\underset{2}{\cancel{6}}\,\cancel{m}}$

$= \dfrac{1 \times 5}{2 \times 2}$

$= \dfrac{5}{4}$

Exercise 2.01 Simplifying algebraic expressions

1 Simplify each expression. Example 1

a $2x^2 + 3x + x^2 + 2x$ **b** $2ab + a + 5ab + b$ **c** $10 - 4m^2 + 6m^2 - 5$

d $3r + 4ar - 2ar - 7r$ **e** $y^2 - y + 3y - 5$ **f** $2a^2 - 4 - a^2 + 6$

g $k^2 - k - 3k - 1$ **h** $3de - 6ed + 2d$ **i** $p^2 - p - p^2 - 5p$

j $-5 + 7u - 10 - 3u$ **k** $t^2 + at + t^2 + 2at$ **l** $15x - 5 + 3x + 5$

2 Simplify each expression. Example 2

a $4ak \times 4am$ **b** $-3d \times 8cd$ **c** $\dfrac{1}{4}t^2w \times 10tw$

d $(5x^3)^2$ **e** $9n^2 \times \dfrac{2n^2}{3}$ **f** $7k^3p \times 2k^2p$

g $-6m \times (-6n)$ **h** $5e \times \dfrac{3e^2}{10}$ **i** $(2xy^2)^2$

j $10ab \times (-2ab)$ **k** $-4n^3 \times \dfrac{1}{4}n^3$ **l** $(-3k)^2$

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Example 3

3 Simplify each expression.

a $\dfrac{4m}{2m}$ **b** $\dfrac{14x^2y}{2xy}$ **c** $-15e^2 \div 3e^2$

d $10u^2t^3 \div 5ut^2$ **e** $\dfrac{3x}{x^3}$ **f** $\dfrac{27pr}{-9pq}$

g $\dfrac{12g^2h}{20gh}$ **h** $a^2 \div a^3$ **i** $\dfrac{15m^4n}{9m^2n^2}$

j $\dfrac{-4de}{20e}$ **k** $u \div 4u$ **l** $6wx^2 \div 24w^2$

4 Which of the following is the simplified expression for $\dfrac{2xy}{5} \times \dfrac{-3}{10y}$? Select **A**, **B**, **C** or **D**.

A $-\dfrac{3x}{25}$ **B** $-\dfrac{3x}{25y^2}$ **C** $-\dfrac{6x}{50}$ **D** $-\dfrac{3}{25y}$

Example 4

5 Simplify each expression.

a $\dfrac{9y}{4} \times 5y$ **b** $\dfrac{4a}{5} \times \dfrac{2h}{7}$ **c** $\dfrac{x}{3} \div \dfrac{x}{6}$

d $\dfrac{20r}{6} \div 4r$ **e** $\dfrac{4nw}{b} \times \dfrac{bn}{2w}$ **f** $\dfrac{3m}{6m} \times \dfrac{10n^2}{4n}$

g $\dfrac{4a}{5} \div \dfrac{2h}{7}$ **h** $\dfrac{2e}{5} \div \dfrac{3e}{8}$ **i** $\dfrac{2de}{7p} \times \dfrac{14p}{4e}$

6 Simplify each expression.

a $2k + 8 - k^2 - 4k$ **b** $ay^2 \times \dfrac{1}{2}y^2$ **c** $-5t \div (-20t)$

d $\dfrac{-32f^3g^2}{4f^2g}$ **e** $-3c^2 + 7c - 7c^2 + 6c$ **f** $4z + z + 8 - 4z^2$

g $-3a \times (-2a^2)$ **h** $\dfrac{24cd}{8de}$ **i** $2 - 4m^2 - 6m^2 - 2$

j $\dfrac{20n^3x}{10nx^2}$ **k** $4x^2 - 2xy + 3x + 4xy$ **l** $6a^4m^2 \div (-2a^2m^2)$

DID YOU KNOW?

Al Gebra is Arabic

In 825 CE, the Persian mathematician **al-Khwarizmi** wrote a book called *Hisab al-jabr w'al-muqabala* (or *The Science of Equations*). The Arabic word '*al-jabr*' meant the process of adding the same amount to both sides of an equation but, when it was translated into Latin, it was changed to 'algebra' and became the name of a whole branch of mathematics, not just equation-solving.

From al-Khwarizmi's name, we get the word *algorithm*. What is the meaning of *algorithm*?

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INVESTIGATION

NUMBER PATTERNS

10	11	12
17	18	19
24	25	26

1. Choose a month from any calendar and draw a box around any block of nine numbers. See the boxed numbers on the right for example.
2. Add together the numbers in any line of three numbers going through the centre of the box (row, column, or diagonal). For example, $10 + 18 + 26$.
3. Add another line of three numbers that goes through the centre of the box.
4. There are two more lines that go through the centre of the box. Find their sums as well.
5. What do you notice? Why does this pattern work? *Hint:* let the number in the top left-hand corner be x.

2.02 Expanding algebraic expressions

Expanding an **expression** means removing grouping symbols (brackets) by multiplying each term inside the grouping symbols by the term outside the grouping symbols.

WS
Expanding algebra

Expanding algebraic expressions

$$a(b + c) = a \times (b + c) = ab + ac$$

EXAMPLE 5

Expand each expression.

a $5(2t - 3)$ **b** $2k(7 + 3k)$ **c** $-3(d^2 - 3d + 8)$

Solution

a $5(2t - 3) = 5 \times 2t - 5 \times 3$

$= 10t - 15$

Multiply each term inside the grouping symbols by the 5 (outside the grouping symbols).

b $2k(7 + 3k) = 2k \times 7 + 2k \times 3k$

$= 14k + 6k^2$

c $-3(d^2 - 3d + 8) = -3 \times d^2 + (-3) \times (-3d) + (-3) \times 8$

$= -3d^2 + 9d - 24$

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EXAMPLE 6

Expand and simplify each expression.

a $x(x-1)+4(x+1)$ **b** $2m(m-3)-(m-6)$ **c** $5u(2-y)+u(2y-9)$

Solution

a $x(x-1)+4(x+1) = x^2 - x + 4x + 4$
$= x^2 + 3x + 4$

Simplify by collecting the like terms $-x$ and $4x$

b $2m(m-3)-(m-6) = 2m^2 - 6m - m + 6$
$= 2m^2 - 7m + 6$

$-(m-6)$ means $-1(m-6)$

c $5u(2-y)+u(2y-9) = 10u - 5uy + 2uy - 9u$
$= u - 3uy$

Exercise 2.02 Expanding algebraic expressions

Example 5

1 Expand each expression.

a $3(a+2)$	**b** $5(3-2b)$	**c** $-2(2a+1)$
d $-6(b-2)$	**e** $3x(x-2)$	**f** $3p(p-a)$
g $-4(2k+4)$	**h** $2t(3-4t)$	**i** $-d(d-5)$
j $k(7-5k)$	**k** $-9b(b-1)$	**l** $2y(7x+4y)$

2 Expand each expression.

a $-6n(4-n)$	**b** $5x(rx+2r)$	**c** $-(2a^2-4)$
d $5b(a^2+3b-7)$	**e** $-(x^2-4x+10)$	**f** $3h(h-7e-4eh)$
g $y(2y+3-y^2)$	**h** $de(d^2-2+e^2)$	**i** $-3v(-3av+v-2a)$

Example 6

3 Expand and simplify each expression.

a $5(x+4)-2(x+3)$	**b** $3(d-4)-2(d+5)$	**c** $6(r+10)-4(r-5)$
d $8(f+2)-(f+7)$	**e** $3(2x-4)-5(3x+4)$	**f** $6x(x+4)-3x(x-1)$
g $3b(b+5)-b(b-8)$	**h** $4w(w-7)-w(w+1)$	**i** $6(k+p)+3(k+2p)$
j $2(a-b)+2(b+a)$	**k** $x(2v+4)-x(v+1)$	**l** $-3(t+w)-2(2t-w)$
m $e(3e+5)-(2e-e^2)$	**n** $-2(a+3)+4(a-3)$	**o** $p(p-q)-q(q-p)$

4 **a** Evaluate $7-5$ and $5-7$. How are the two answers related?

b Evaluate $4-10$ and $10-4$.

c Is $a-b$ always the same as $-(b-a)$? Can you prove it algebraically?

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INVESTIGATION

MENTAL MULTIPLICATION BY EXPANDING

Expanding is useful for multiplying numbers mentally without using a calculator, especially if one of the numbers is close to 10, 100 or 1000.

1 Study the following examples.

a $35 \times 11 = 35 \times (10 + 1)$ Think of 11 as 10 + 1

$= 35 \times 10 + 35 \times 1$ Expand

$= 350 + 35$ Simplify

$= 385$

b $43 \times 102 = 43 \times (100 + 2)$ Think of 102 as 100 + 2

$= 43 \times 100 + 43 \times 2$ Expand

$= 4300 + 86$ Simplify

$= 4386$

c $16 \times 8 = 16 \times (10 - 2)$ Think of 8 as 10 – 2

$= 16 \times 10 - 16 \times 2$ Expand

$= 160 - 32$ Simplify

$= 128$

2 Use expanding to multiply each pair of numbers mentally.

a 25×12 **b** 18×9 **c** 6×105

d 87×11 **e** 50×99 **f** 45×8

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2.03 Formulas

Substitution code puzzle

EXAMPLE 7

If $x = 4$, $y = 7$ and $z = -1$, evaluate each expression.

a $5x^2 + 2z$ **b** $\dfrac{z-8}{10-y}$ **c** $6(yz + 5)$ **d** $\sqrt{3y - x - 1}$

Solution

a $5x^2 + 2z = 5 \times 4^2 + 2 \times (-1)$
$= 78$

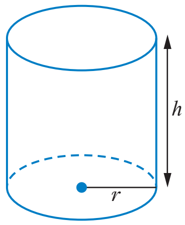
b $\dfrac{z-8}{10-y} = \dfrac{-1-8}{10-7}$
$= -3$

c $6(yz + 5) = 6(7 \times (-1) + 5)$
$= 6 \times (-2)$
$= -12$

d $\sqrt{3y - x - 1} = \sqrt{3 \times 7 - 4 - 1}$
$= \sqrt{16}$
$= 4$

Formulas

A **formula** is an algebraic **rule** that describes a mathematical relationship between variables. For example, the volume of a cylinder has the formula $V = \pi r^2 h$, where r is the radius of the cylinder's base and h is its perpendicular height.



Because the formula describes V, with V on the left-hand side of the '=' sign, we say that V is the **subject** of the formula.

EXAMPLE 8

If a principal, $\$P$, is invested at an interest rate of r **per annum** (where r is written as a decimal) then, after n years, it will grow to $\$A$, where A is given by the compound interest formula $A = P(1 + r)^n$. Use the formula to calculate (to the nearest cent) the final amount of an investment of \$4000 after 6 years at 11% p.a.

Solution

$P = 4000$, $r = 11\% = 0.11$, $n = 6$

$A = P(1 + r)^n$
$= 4000(1 + 0.11)^6$
$= 7481.658\,209\ldots$
≈ 7481.66

The final amount is \$7481.66.

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EXAMPLE 9

The correct **dosage** of a medicine for infants (babies up to 2 years) depends on the child's age or weight. There are different formulas for calculating this.

Fried's rule: $\text{Child dosage} = \dfrac{\text{age in months}}{150} \times \text{adult dosage}$

Young's rule: $\text{Child dosage} = \dfrac{\text{age in years}}{\text{age in years} + 12} \times \text{adult dosage}$

Ryan is 18 months old and weighs 13 kg. The adult dosage of a drug prescribed for him is 20 g. Calculate Ryan's dosage using:

a Fried's rule

b Young's rule.

Solution

a $$\begin{aligned}\text{Ryan's dosage} &= \frac{18}{150} \times 20 \text{ g} \\ &= 2.4 \text{ g}\end{aligned}$$

b $$\begin{aligned}\text{Ryan's dosage} &= \frac{1.5}{1.5 + 12} \times 20 \text{ g} \\ &= 2.2222\ldots \text{ g} \\ &\approx 2.2 \text{ g}\end{aligned}$$

Age = 18 months = 1.5 years

EXAMPLE 10

From a height of h metres above sea level, an observer can see a distance of d km to the horizon, where $d = 8\sqrt{\dfrac{h}{5}}$. What distance, correct to the nearest kilometre, can be seen from the top of Sydney Harbour Bridge, 134 m above sea level?

Solution

$$\begin{aligned} h &= 134 \\ d &= 8\sqrt{\frac{h}{5}} \\ &= 8\sqrt{\frac{134}{5}} \\ &= 41.414\,97\ldots \\ &\approx 41 \text{ km}\end{aligned}$$

A distance of 41 km can be seen from the top of Sydney Harbour Bridge.

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Exercise 2.03 Formulas

Example 7

1 If $a = -3$, $b = 10$ and $c = 6$, evaluate each expression.

a $b^2 - a^2$ **b** $\frac{b+c}{ac}$ **c** $\sqrt{c+3b}$ **d** $4(3a+9)$

e $8c + 4a$ **f** $\frac{2b}{5}$ **g** $b(b-4)$ **h** $\sqrt{\frac{9b}{c+4}}$

i $c^2 + c$ **j** $\frac{2ac}{3}$ **k** $\sqrt{a^2+7c-2}$ **l** $(a-b)^2$

2 Calculate the volume, correct to two decimal places, of a cylinder with a base radius of 4.07 cm and perpendicular height of 11.58 cm, using the formula $V = \pi r^2 h$.

3 The temperature T (in °C) of the water in a kettle t minutes after it is switched on is given by the formula $T = 18t + 28$. Find the temperature of the water:

a 4 minutes after it is turned on

b $1\frac{1}{2}$ minutes after it is turned on

c when the kettle is first turned on.

4 The formula for converting Australian dollars ($A) to US dollars ($US) is US = 0.722A. Convert the following $A amounts to $US, correct to the nearest cent.

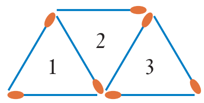
a $20.00 **b** $89.50 **c** $4800

5 If an object is moving with speed u m/s and acceleration a m/s^2, then its speed v m/s after t seconds is $v = u + at$.

a Which variable is the subject of the formula?

b Calculate the speed of a car after 5 seconds if its speed now is 6 m/s and it is accelerating at 2 m/s^2.

6 The number of matches, m, needed to make this pattern of triangles is $m = 2t + 1$, where t is the number of triangles in the pattern. How many matches are required to make:



a 8 triangles?

b 40 triangles?

c 150 triangles?

7 The volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$. Calculate, correct to one decimal place, the volume of a sphere with radius 14.5 cm.

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8 The time, T seconds, it takes a swing to go back and forth once is $T = 2\pi\sqrt{\frac{l}{g}}$ where l m is the length of the swing and g is the gravitational acceleration. Find T, correct to two decimal places, if $l = 2.35$ and $g = 10$.

Shutterstock.com/Stephen Rees

9 Use the compound interest formula $A = P(1 + r)^n$ to calculate the amount to which a principal of \$5600 will grow if invested at 9.4% p.a. for 5 years.

Example 8

10 The maximum distance, d m, a ball travels if thrown with speed v m/s, is $d = \frac{v^2}{g}$. Find the maximum distance if a ball is thrown at a speed of 11.5 m/s and $g = 9.8$.

11 The body mass index (BMI) of an adult is $B = \frac{m}{h^2}$, where m is the mass in kilograms and h is the height in metres. Zoe is 1.7 m tall and weighs 60 kg.

a What is the subject of the formula given above?

b Calculate Zoe's BMI, correct to one decimal place.

c If a BMI between 21 and 25 is an indication of good health, then how can Zoe improve her health?

12 The formula for converting Fahrenheit temperatures (°F) to Celsius (°C) is $C = \frac{5}{9}(F - 32)$.
Convert each of the following temperatures to Celsius, correct to the nearest degree.

a New York 77°F

b Los Angeles 100°F

c Rio de Janeiro 59°F

d Normal body temperature 98.6°F

13 The formula for converting a speed of k km/h to metres per second (m/s) is $M = \frac{5k}{18}$.
A speed of 80 km/h is closest to which of the following speeds? Select **A**, **B**, **C** or **D**.

A 22 m/s **B** 32 m/s **C** 35 m/s **D** 47 m/s

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Example 9

14 Mia is two years old and weighs 11 kg. A nurse calculates her medicine dosage given the adult dosage is 900 mg per day. Calculate Mia's daily dosage to the nearest mg using:

a Fried's rule **b** Young's rule.

15 Clark's rule for children 2 years and over is:

$$\text{Child dosage} = \frac{\text{weight in kg}}{70} \times \text{adult dosage}$$

Ashleigh is 16 years old and weighs 50 kg. She takes a drug with a recommended adult dose of 1680 mg each day. Using Clark's rule, what would be Ashleigh's daily dosage? Select **A**, **B**, **C** or **D**.

A 1200 mg **B** 1800 mg **C** 300 mg **D** 336 mg

16 Cowling's rule for calculating a child's dosage is:

$$\text{Child dosage} = \frac{\text{age in years} + 1}{24} \times \text{adult dosage}$$

If the adult daily dosage of an antibiotic is 100 mg, use Cowling's rule to find the daily dosage, correct to one decimal place, for:

a Brittany, aged 2 **b** Patrick, aged 6 **c** Mercedes, aged $10\frac{1}{2}$.

17 Elena is $1\frac{1}{2}$ years old. If the adult dosage of a drug is 1000 mg per day, find, correct to the nearest mg, Elena's appropriate daily dosage using:

a Young's rule **b** Fried's rule.

Example 10

18 The distance, d km, an observer can see to the horizon from a height, h m, above sea level is $d = 8\sqrt{\frac{h}{5}}$. What distance (correct to the nearest kilometre) can a person see from the top of Sydney Tower at 305 m?

19 Liam earns a weekly wage of \$450 plus commission of 11% of the value of phone cards he sells in excess of \$900. His weekly pay is given by the formula $P = 450 + 0.11(V - 900)$, where V is the total value of the phone cards sold. How much will Liam earn for selling \$2310 worth of phone cards in a week?

20 The braking distance (in metres) of a bicycle travelling at a speed of V m/s is $d = \frac{V(V+1)}{2}$. Calculate the braking distance of a bicycle travelling at a speed of 6 m/s.

21 The surface area of a cylinder of radius, r, and height, h, is $S = 2\pi r(r + h)$. Calculate the surface area, correct to two decimal places, of a cylinder with radius 3 cm and height 8 cm.

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22 The speed, V m/s, required for a spacecraft to escape Earth's gravitational pull during take-off is $V = \sqrt{2gr}$ where g is 9.8 m/s^2 and r is the radius of Earth (6 378 000 m). Which of the following is closest to the escape speed of a spacecraft leaving Earth's atmosphere? Select **A**, **B**, **C** or **D**.

A 8840 m/s **B** 11 180 m/s **C** 12 500 m/s **D** 35 000 m/s

DID YOU KNOW?

The body mass index

The **body mass index (BMI)** is used by the World Health Organization (WHO) as an international standard of health and fitness for adults. The formula for BMI is $B = \frac{m}{h^2}$, where m is a person's mass in kilograms and h is the person's height in metres. This is a convenient measure that represents the health of an adult by a single value. The table describes a range of BMI scores.

BMI	Health status
Under 18.5	Underweight
18.5 to 24.9	Normal
25.0 to 29.9	Overweight
30 and above	Obese

According to this model, about 38% of Australian adults are overweight and 28% of Australian adults are obese, having a greater risk of heart disease, stroke, diabetes, high blood pressure and high cholesterol.

Because the BMI is an algebraic *model*, the WHO acknowledges that it has some limitations and inaccuracies. It does not take into account a person's frame size, muscle mass, bone density or distribution of body fat. For this reason, the BMI should not be applied to bodybuilders, athletes, children under 19, pregnant women, the frail and sedentary elderly, and Aboriginal, Pacific Island and Asian people.

- **Why is the BMI less accurate for measuring the levels of fitness of the types of people listed above?**
- **Would each of the types of people listed above score unusually high or unusually low on the BMI scale?**

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INVESTIGATION

GETTING THE RIGHT FORMULA

Listed below are 15 commonly used formulas. As a group or individual activity, select eight formulas and, for each one:

a describe what the formula is used for

b write the subject of the formula, and what it stands for

c describe what the other variables in the formula stand for.

1	$V = \frac{1}{3}Ah$	**2**	$A = 180(n - 2)$	**3**	$c^2 = a^2 + b^2$
4	$A = \pi r^2$	**5**	$S = 2\pi r^2 + 2\pi rh$	**6**	$I = Prn$
7	$m = \frac{y_2 - y_1}{x_2 - x_1}$	**8**	$C = 2\pi r = \pi d$	**9**	$A = \frac{1}{2}xy$
10	$S = \frac{d}{t}$	**11**	$A = \frac{1}{2}(a+b)h$	**12**	$A = P(1 + r)^n$
13	$A = s^2$	**14**	$V = \pi r^2 h$	**15**	$V = \frac{4}{3}\pi r^3$

Equations code puzzle

Solving equations by balancing

Solving equations by backtracking

Solving equations using diagrams

2.04 Solving equations

An **equation** contains an algebraic expression and an equals (=) sign. For example, $3x - 4$ is an **expression,** while $3x - 4 = -13$ is an **equation**. An equation is **solved** when the value of the variable (for example, x) is found that makes the equation true.

Solving equations

- Keep the equation balanced by doing the same operation on both sides.
- Aim to have the variable (for example, x) on one side of the equation and a number on the other side. For example, $x = 4$.

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EXAMPLE 11

Solve each equation.

a $3x - 4 = -13$ **b** $\frac{b}{2} + 7 = 1$ **c** $\frac{h-10}{4} = 3$

Solution

a

$$3x - 4 = -13$$

$$3x - 4 + 4 = -13 + 4 \qquad \text{Adding 4 to both sides}$$

$$3x = -9$$

Remember: We are aiming to get x on its own.

$$\frac{3x}{3} = \frac{-9}{3} \qquad \text{Dividing both sides by 3}$$

$$x = -3$$

Check by substituting $x = -3$ back into the original equation:

$$\begin{aligned} \text{LHS} &= 3(-3) - 4 \\ &= -13 \\ &= \text{RHS} \end{aligned}$$

LHS means 'left-hand side'.

RHS means 'right-hand side'.

b

$$\frac{b}{2} + 7 = 1$$

$$\frac{b}{2} + 7 - 7 = 1 - 7 \qquad \text{Subtracting 7 from both sides}$$

$$\frac{b}{2} = -6$$

$$\frac{b}{2} \times 2 = -6 \times 2 \qquad \text{Multiplying both sides by 2}$$

$$b = -12$$

Check by substituting $b = -12$ back into the original equation:

$$\begin{aligned} \text{LHS} &= \frac{-12}{2} + 7 \\ &= 1 \\ &= \text{RHS} \end{aligned}$$

c

$$\frac{h-10}{4} = 3$$

$$\frac{h-10}{4} \times 4 = 3 \times 4 \qquad \text{Multiplying both sides by 4}$$

$$h - 10 = 12$$

$$h - 10 + 10 = 12 + 10 \qquad \text{Adding 10 to both sides}$$

$$h = 22$$

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EXAMPLE 12

Solve each equation. ← These equations need more steps to solve.

a $5y + 5 = 2y + 17$

b $\dfrac{2m-7}{3} = 6$

c $4(1 - 2t) = 16$

Solution

a $5y + 5 = 2y + 17$ ← Remember: We are aiming to get y on its own.

$5y + 5 - \mathbf{2y} = 2y + 17 - \mathbf{2y}$ Subtracting $2y$ from both sides to get all the y terms on the LHS

$3y + 5 = 17$

$3y + 5 - \mathbf{5} = 17 - \mathbf{5}$ Subtracting 5 from both sides to get all the numbers on the RHS

$3y = 12$

$\dfrac{3y}{3} = \dfrac{12}{3}$ Dividing both sides by 3

$y = 4$

Checking:

LHS $= 5(4) + 5$

$= 25$

RHS $= 2(4) + 17$

$= 25$

$=$ LHS

b $\dfrac{2m-7}{3} = 6$

$\dfrac{2m-7}{3} \times 3 = 6 \times 3$ Multiplying both sides by 3

$2m - 7 = 18$

$2m = 25$ Adding 7 to both sides

$m = \dfrac{25}{2}$ Dividing both sides by 2

$= 12\frac{1}{2}$

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Checking:

$$\text{LHS} = \frac{2\left(12\frac{1}{2}\right)-7}{3}$$
$$= \frac{18}{3}$$
$$= 6$$
$$= \text{RHS}$$

c $4(1 - 2t) = 16$

$4 - 8t = 16$ Expanding LHS first

$-8t = 12$ Subtracting 4 from both sides

$t = \frac{12}{-8}$ Dividing both sides by (−8)

$= -1\frac{1}{2}$

Exercise 2.04 Solving equations

Example 11

1 Solve each equation.

a $3d + 2 = 20$ **b** $2p - 3 = 2$ **c** $4u + 6 = 20$

d $5a + 3 = -12$ **e** $12b + 8 = 4$ **f** $3 - 2a = -6$

g $3m = m - 10$ **h** $\frac{3h}{4} = 9$ **i** $\frac{r-1}{6} = 2$

j $-\frac{2x}{5} = 8$ **k** $\frac{y+2}{-3} = 1$ **l** $11 - 4n = 15$

m $5y + 6 = 4y + 11$ **n** $\frac{4c}{10} = 3$ **o** $\frac{z}{3} - 11 = 9$

2 What is the solution to $2t - 4 = 10 + t$? Select **A**, **B**, **C** or **D**.

A $t = 3$ **B** $t = 6$ **C** $t = 4\frac{2}{3}$ **D** $t = 14$

Example 12

3 Solve each equation.

a $5k - 13 = 3k + 9$ **b** $8e = 2(e - 6)$ **c** $\frac{2f+7}{2} = 10$

d $3(x - 2) = 45$ **e** $\frac{w}{5} - 8 = 6$ **f** $4(2d - 9) = -12$

g $\frac{4n+7}{9} = 2$ **h** $7u + 7 = 2u - 10$ **i** $3p + 4 = 4p$

j $\frac{5z+8}{6} = -3$ **k** $\frac{8-2b}{2} = 7$ **l** $6(2 - 3q) = -24$

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4 In which line was an error made in solving the following equation? Select **A**, **B** or **C**.

A Line 2 **B** Line 3 **C** Line 4

$$\begin{aligned}
\text{Line 1:}\quad \frac{c-4}{8}+2 &= 6\\
\text{Line 2:}\quad \frac{c-4}{8} &= 8\\
\text{Line 3:}\quad c-4 &= 64\\
\text{Line 4:}\quad c &= 68
\end{aligned}$$

2.05 Formulas and equations

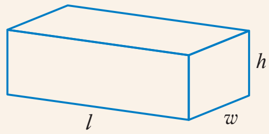
WS Working with formulas

Sometimes, after we **substitute** values into a formula, the result is an equation that must be solved.

EXAMPLE 13

The surface area of a rectangular prism of length l, width w and height h is $S = 2lw + 2lh + 2wh$.

If a rectangular prism with surface area 132 cm^2 has length 7 cm and width 3 cm, find its height.



Solution

$$\begin{aligned}
S &= 132, l = 7, w = 3\\
S &= 2lw + 2lh + 2wh\\
132 &= (2 \times 7 \times 3) + (2 \times 7 \times h) + (2 \times 3 \times h) && \text{Substituting into formula}\\
132 &= 42 + 14h + 6h\\
132 &= 42 + 20h && \text{Collecting like terms}\\
90 &= 20h && \text{Subtracting 42 from both sides}\\
h &= \frac{90}{20} && \text{Dividing both sides by 20}\\
&= 4\frac{1}{2}
\end{aligned}$$

The height of the rectangular prism is $4\frac{1}{2}$ cm.

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EXAMPLE 14

Formulas and equations

The formula for converting Fahrenheit temperature (°F) to Celsius (°C) is $C = \frac{5}{9}(F - 32)$. Convert 100°C to °F.

Solution

When $C = 100$:

$$100 = \frac{5}{9}(F - 32)$$

$$\frac{5}{9}(F - 32) = 100$$

$$5(F - 32) = 900 \qquad \text{Multiplying both sides by 9}$$

$$5F - 160 = 900 \qquad \text{Expanding the LHS}$$

$$5F = 1060 \qquad \text{Adding 160 to both sides}$$

$$F = \frac{1060}{5} \qquad \text{Dividing both sides by 5}$$

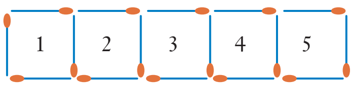
$$= 212$$

100°C = 212°F.

Exercise 2.05 Formulas and equations

Example 13

1 The number of matchsticks, m, needed to make a pattern of s squares is $m = 3s + 1$.



a How many matches are needed to make:
i 4 squares? **ii** 10 squares?

b How many squares can be made from:
i 22 matches? **ii** 55 matches?

2 Find the base length, b, of a triangle with area 90 cm^2 and perpendicular height 15 cm, given the area formula $A = \frac{1}{2}bh$.

3 The average speed of a moving object in metres per second is $s = \frac{d}{t}$, where d is the distance travelled in metres and t is the time taken in seconds.

a Find the distance travelled by a car in 20 seconds if its speed is 15 m/s.

b Find the time taken (to the nearest second) for a cyclist to travel 250 m if his speed is 4.2 m/s.

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Example 14

4 The formula for the area of a trapezium is $A = \frac{h}{2}(a + b)$, where a and b are the lengths of the parallel sides, and h is the distance between them. What is the length of one parallel side of a trapezium if the other parallel side is 7 m, the distance between them is 5 m and the area is 22.5 m^2?

5 The mean, M, of three numbers, x, y and z, is calculated using the formula $M = \frac{x + y + z}{3}$. If three numbers have a mean of 17 and two of the numbers are 10 and 20, find the third number.

6 The circumference of a circle with radius r is $C = 2\pi r$. If a circle has a circumference of 50.27 cm, find its radius, correct to the nearest centimetre.

7 If a principal, $\$P$, is invested at an interest rate of r per annum (where r is written as a decimal) then, after n years, it will grow to $\$A$, where A is given by the compound interest formula $A = P(1 + r)^n$. What principal needs to be invested at 11% p.a. for it to grow to \$6000 in 3 years? Express your answer to the nearest cent.

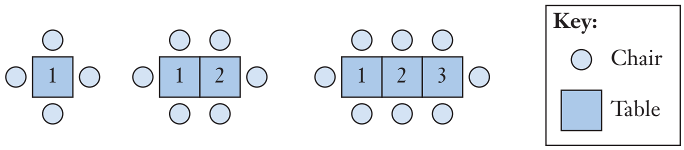
8 A kettle is boiled and the temperature, T°C, of the water after t minutes is $T = 18t + 28$. After how many minutes is the temperature:

a 64°C? **b** 92.8°C?

9 The formula for converting miles (M) to kilometres (K) is $K = 1.61M$.
Convert each of the following distances to miles, correct to two decimal places.

a 5 km **b** 1.5 km

10 The number of chairs, c, that can be seated around t square tables is $c = 2t + 2$.



a How many chairs can be seated around:
i 5 tables? **ii** 12 tables?

b How many square tables are required to seat:
i 24 chairs? **ii** 40 chairs?

11 Rhianna earns a weekly wage of \$540 plus commission of 12% on the value of cosmetics she sells in excess of \$2000. Her total pay, P, is given by the formula $P = 540 + 0.12\ (V - 2000)$, where V is the value of the cosmetics sold that week. What was the value of the cosmetics Rhianna sold if her total pay was \$852?

12 The surface area of a rectangular prism of length l, width w and height h is $S = 2lw + 2lh + 2wh$. If a rectangular prism has length 4 cm, width 2 cm and surface area 58 cm^2, what is its height? Select **A**, **B**, **C** or **D**.

A 3.5 cm **B** $4\frac{1}{6}$ cm **C** 5.5 cm **D** 7.25 cm

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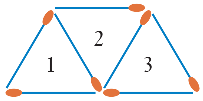
13 The formula for converting k kilometres to M miles is $M = \frac{5k}{8}$. Convert 12 miles to kilometres.

14 The number of matchsticks, m, needed to make this pattern of triangles is $m = 2t + 1$, where t is the number of triangles in the pattern.



How many triangles can be made with:

a 37 matches? **b** 55 matches?

15 For babysitting, Kirstie charges \$18 for the first hour and \$14 for each hour after that. This charge, \$$C$, can be expressed using the formula $C = 18 + 14(h - 1)$, where h is the number of hours worked. If Kirstie earned \$46 for babysitting one night, for how many hours did she work?

16 Mohammed works in a high-rise office building on the 32nd floor. At lunchtime, he travels down in the lift at a speed of two floors per second. This is described by the formula $F = 32 - 2t$ where F is the floor being passed by the lift and t is the time in seconds.

a Which floor is Mohammed passing after:
  i 8 seconds? **ii** 14 seconds?

b After how many seconds will Mohammed pass the:
  i 18th floor? **ii** 8th floor?

17 According to one theory, the recommended nightly hours of sleep for a child is $S = 8 + \frac{18 - a}{2}$, where a is the age of the child in years. What is the age of a child who requires $14\frac{1}{2}$ hours of sleep each night? Select **A**, **B**, **C** or **D**.

A 2.5 years **B** 3 years **C** 3.5 years **D** 5 years

18 If an object has an initial speed u m/s and acceleration a m/s^2 after t seconds, its final speed (in m/s) is given by the formula $v = u + at$. Find the acceleration of an object if its initial speed is 8 m/s and its final speed after 12 seconds is 44 m/s.

Changing the subject of a formula

2.06 Changing the subject of a formula

In the formula $v = u + at$, v is called the **subject** of the formula because it is on the LHS. To change the subject of a formula to another variable, use the same rules as for solving an equation. The answer is not a number but an algebraic equation (another formula).

EXAMPLE 15

If a moving object has initial speed u m/s and acceleration a m/s^2, its final speed v m/s is given by the formula $v = u + at$. Change the subject of this formula to:

a u **b** t.

Solution

a

$v = u + at$	
$u + at = v$	Swap sides so new subject u appears on LHS
$u = v - at$	Subtract at from both sides

b

$v = u + at$	
$u + at = v$	Swap sides so new subject t appears on LHS
$at = v - u$	Subtract u from both sides
$t = \dfrac{v - u}{a}$	Divide both sides by a

EXAMPLE 16

The volume of a cylinder is given by the formula $V = \pi r^2 h$, where r is the radius of the circular base and h is the height. Make h the subject of the formula.

Solution

$V = \pi r^2 h$	
$\pi r^2 h = V$	Swap sides so new subject h appears on LHS
$h = \dfrac{V}{\pi r^2}$	Divide both sides by πr^2

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Exercise 2.06 Changing the subject of a formula

Example 15

1 Make x the subject of each formula.

a $y = 2x + 4$ **b** $T = 3x - 7$ **c** $d = \frac{x+1}{3}$

d $p = 18 - x$ **e** $k = 4x + r$ **f** $C = n + \frac{x}{2}$

g $S = 10bx$ **h** $V = \frac{x-5}{6}$ **i** $z = 12 - ax$

2 If an object travels a distance, d, in time, t, its average speed, S, is given by the formula $S = \frac{d}{t}$. Change the subject of the formula to:

a d **b** t.

3 The angle sum of a shape with n sides is $A°$, where $A = 180(n - 2)$.

a Use the formula to find the angle sum of a shape with 12 sides.

b Make n the subject of the formula.

c If the angle sum of a polygon is 1260°, how many sides does it have?

4 The volume of a **pyramid** has the formula $V = \frac{1}{3}Ah$, where A is the area of the base and h is the perpendicular height. Which of the following is the correct formula for A? Select **A**, **B**, **C** or **D**.

A $A = \frac{1}{3}Vh$ **B** $A = \frac{3}{Vh}$ **C** $A = 3Vh$ **D** $A = \frac{3V}{h}$

5 The area of a trapezium has the formula $A = \frac{h}{2}(a + b)$. Change the subject to:

a h **b** b.

Example 16

6 Change the subject of each formula to the variable shown in brackets.

a $A = \pi r^2$ [π] **b** $y = mx + c$ [m] **c** $v^2 = u^2 + 2as$ [s]

d $z = \frac{x-m}{s}$ [x] **e** $E = mc^2$ [m] **f** $K = \frac{1}{2}mv^2$ [m]

g $A = P(1 + r)^n$ [P] **h** $x = 6y + 3$ [y] **i** $A = \frac{1}{2}bh$ [h]

j $T^2 = \frac{4\pi^2 l}{g}$ [l] **k** $V = IR - E$ [R] **l** $e = ir + \frac{Q}{C}$ [Q]

m $s = ut + \frac{1}{2}at^2$ [a] **n** $K = \frac{m+d}{w}$ [d]

7 The body mass index of an adult is given by the formula $B = \frac{m}{h^2}$, where m is mass in kilograms and h is in metres. Change the subject of the formula to m and hence find, to the nearest kilogram, the mass of a person with a body mass index of 25 and a height of 1.55 m.

8 The volume of a cone is $V = \frac{1}{3}\pi r^2 h$, where r is the radius of the base and h is the height. Show that this formula can be rewritten as $h = \frac{3V}{\pi r^2}$ and hence find (to the nearest centimetre) the height of a cone with volume 3539.53 cm^3 and base radius 13 cm.

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9 Make F the subject of the formula $C = \frac{5}{9}(F - 32)$ and hence find F when:

a $C = 40$ **b** $C = 100$.

10 Make P the subject of the formula $M = \frac{10}{\pi h P}$, then find the value of P, correct to two significant figures, if $M = 9.8$ and $h = 0.27$.

SAMPLE HSC PROBLEM

Paula the plumber charges according to the formula $C = 60 + 32(h - 1)$, where $\$C$ is the charge and h is the number of hours worked.

a How much does Paula charge for working $2\frac{1}{2}$ hours?

b Make h the subject of the formula.

c Find the number of hours Paula worked if she charged $172.

Study tip

Topic summaries and mind maps

Summarise each topic once you have completed it, to create useful study notes for revising the course, especially before exams. Use a notebook or folder for listing the important ideas, formulas, terminology and skills of each topic. Educational research shows that effective learning takes place when we rewrite learned knowledge in our own words.

A good topic summary runs for two to four pages. It is a condensed, personalised version of your course notes. This is your interpretation of a topic, so include your own comments, symbols, diagrams, observations and reminders. Highlight important facts using boxes and include a glossary of key words and phrases. (There is a glossary at the back of this book.)

A mind map is a topic summary in graphic form, with boxes, branches and arrows showing the connections between the main ideas of the topic. (See the next page for an example.) The topic name is written at the centre of the map, with branches leading off to key concepts or subheadings. Smaller branches hang off these branches, listing important details and formulas. Mind maps are powerful because they present an overview of a topic on one large sheet of paper. Visual learners absorb and recall information better using mind maps.

When compiling a topic summary, use your class notes and this textbook, especially the front and back sections of each chapter. Ask your teacher for a copy of the course syllabus or the school's teaching program, which lists the knowledge and skills of every topic in dot point form.

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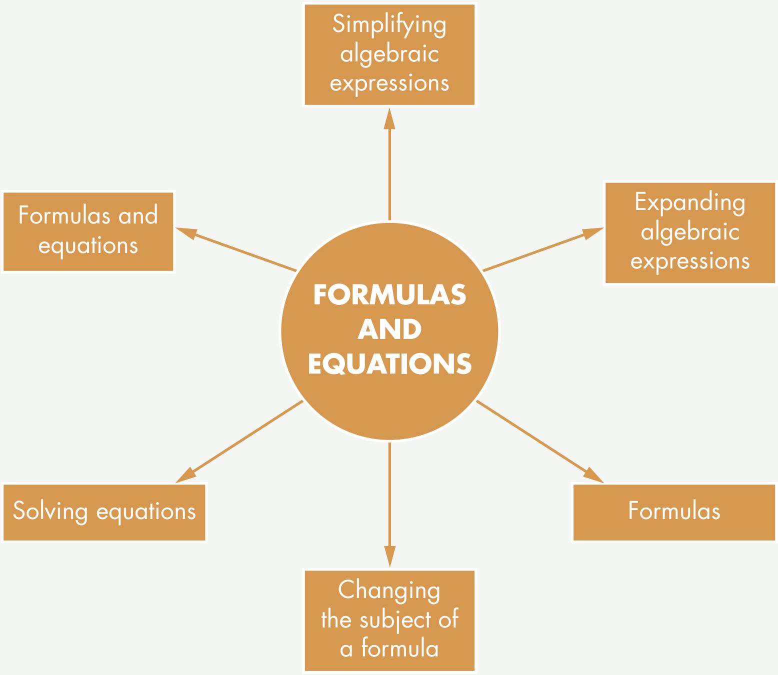
2. CHAPTER SUMMARY

This chapter, Formulas and equations, revised basic and extended algebra skills. Make sure you master the algebraic techniques required to:

WS Algebra review

- simplify expressions
- expand expressions
- solve equations
- work with formulas
- change the subject of a formula.

Make a summary of this topic. Use the chapter outline at the beginning of this chapter and the mind map below as a guide. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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2. TEST YOURSELF

Exercise 2.01

1 Simplify each expression.

a $5ut + 2t^2 - t^2 + 4ut$

b $3k^2 \times 5k$

c $-9d \times \frac{2d}{3}$

d $8p^2 \times 4p^3$

e $-1 + 4h + 8 - 10h$

f $\frac{16r^3}{2r}$

g $(-3d^2)^3$

h $\frac{9n^2}{15n^2}$

i $-3v^2w^2 \div 21vw$

j $10x^2 + 7x - 2x^2 + x$

k $\frac{24bc}{8b^2}$

l $\frac{-a}{a^3}$

m $\frac{48r}{6} \div 4r$

n $\frac{4y}{3} \times \frac{5v}{10}$

o $\frac{10x}{18p} \times \frac{3p}{20}$

p $\frac{3y}{2a} \div \frac{9dy}{10d}$

Exercise 2.02

2 Expand each expression.

a $5(2x - 4)$

b $-3(a + 7)$

c $4(12t - y)$

d $-9(r^2 + 2w)$

e $8mn(m - n)$

f $-2d(4d - d^2)$

Exercise 2.02

3 Expand and simplify each expression.

a $3(4x + 1) + 2(x - 2)$

b $2n(n - 1) + (n - 1)$

c $6(2 - d) - 4(d - 3)$

d $p(p + 4) - p(p + 8)$

e $3(4u + 5) - (u + 7)$

f $h(5h - 1) + 3h(h + 9)$

Exercise 2.03

4 If $p = 4$, $q = -5$ and $r = 20$, then evaluate each expression.

a $3p^2 + 4r$

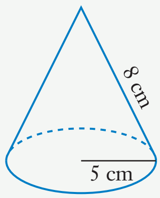
b $\frac{7r}{q}$

c pqr

d $\sqrt{p(r - q)}$

Exercise 2.03

5 The surface area of a cone is given by the formula $S = \pi r(r + s)$, where r is the radius of the base and s is the slant height of the cone. Find, correct to two decimal places, the surface area of a cone with base radius 5 cm and slant height 8 cm.



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6 Brett earns a weekly wage of \$480 plus a commission of 10% of the value of the insurance plans he sells in excess of \$1200. His total weekly pay, \$$P$, is given by the formula $P = 480 + \frac{V - 1200}{10}$, where V is the value of the insurance plans sold. Calculate Brett's pay for a week in which he sold \$3400 worth of insurance plans.

Exercise 2.03

7 Solve each equation.

a $5p - 4 = 21$ **b** $-2a + 6 = 8$ **c** $\frac{b-3}{2} = -6$

d $23 - 8r = 19$ **e** $\frac{4n}{5} = 11$ **f** $\frac{r}{3} + 7 = 1$

g $\frac{20-4n}{4} = 7$ **h** $3t + 13 = t - 12$ **i** $5(2g - 4) = -30$

Exercise 2.04

8 If an object is travelling with initial speed u m/s, accelerating at a rate of a m/s^2, and covers a distance s m, then its final speed v m/s follows the rule $v^2 = u^2 + 2as$. Calculate the distance travelled by a car whose speed increases from 11 m/s to 28 m/s with an acceleration of 3 m/s^2.

Exercise 2.05

9 According to one theory, the formula that links the surrounding air temperature, T°C, to the number of chirps per minute, C, made by a cricket at night during summer is $T = \frac{C}{8} + 3$. How many chirps per minute are made by the cricket when the temperature is 13°C?

Exercise 2.05

10 The average blood pressure, P, of a person aged y years, measured in millimetres of mercury (mmHg), is given by the formula $P = 110 + \frac{y}{2}$. Make y the subject of this formula and use it to find the age of a person whose blood pressure is 124 mmHg.

Exercise 2.06

Chapter quiz