



Statistics and probability **Probability**

Many people suffer from aviophobia, the fear of flying. However, we are more likely to be involved in a car accident than a plane crash. There is a 1-in-5000 chance of being killed on a car trip compared to 1-in-11 million on a plane flight. This is amazing when you consider that each day worldwide there are 27 000 planes in the sky carrying over 4 million passengers. Why do you think there are so few plane accidents compared to road accidents? And why are people more afraid of flying than travelling in a car?

NEW CENTURY MATHS 7



Chapter outline

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11-03 The range of probability	U	F	PS	R
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Wordbank

certain Sure to happen; the opposite of impossible

complementary event The 'opposite' event; for example, the complementary event to selecting an Ace from a deck of cards is *not* selecting an Ace

equally likely To have exactly the same chance of occurring; for example, when tossing a coin, heads or tails are equally likely

event A result involving one or more outcomes; for example, the event of rolling an even number on a die contains the three outcomes {2, 4, 6}

outcome A single result from a situation or experiment; for example, one outcome when rolling a die is to roll a 6

random Describes a situation in which every possible outcome has an equal chance

sample space The set of all possible outcomes of a situation or experiment

		In this chapter you	will:						
Maths clip Sample spaces MAT07SPVT00009 Weblink Master class on probability	• • • •	construct sample spaces for single-step experiments with equally likely outcomes assign probabilities to the outcomes of events and determine probabilities for events ecognise that probabilities range from 0 to 1 compare observed frequencies across experiments with expected frequencies dentify complementary events and use the sum of probabilities to solve problems							
		SkillCheck							
Worksheet StartUp assignment 11 MAT07SPWK10086 Worksheet Chance cards MAT07SPWK10087 Describing probabilities MAT07SPWK00061	1 2 3 4 5	Rate each event as being in a Choosing a pink ball from b It will rain next month. c A coin shows tails where d You obtain a driver's like e You will have your drive f Rolling two sixes on a period g You are over 10 years of h You draw a red card from i You will use a computer j It will snow in your tower Simplify each fraction. a $\frac{2}{10}$ b Convert each number to a a $\frac{7}{8}$ b Convert each number to a a 0.6 b Is the chance of each event a You having another brow b You going overseas this c You being a parent in 2 e You getting a good reput f Your home phone ring a Draw an interval 15 cm impossible to certain. no way not likely i the state of	npossible, unlikely om a bag of blue h a you toss it. cence tomorrow. er's licence in 20 your of dice ld. om a deck of play. r today. rn tomorrow 8 18 decimal. 3 5 percentage: 0.31 c more than or less other or sister one year st Monday 0 years time ort for English thin ng today long and use it to ever	y, even chance, likely or oballs years time. ing cards. $c \frac{27}{30}$ c 26% $c \frac{3}{4}$ $s \tanh \frac{1}{2}?$ day s year p make a probability scal	certain. d $\frac{15}{80}$ d 70% d $\frac{1}{20}$ e for the chances from almost definitely				
		0 impossible		$\frac{1}{2}$	1 certain				

blue

green

red

yellow

- **b** On your scale, mark the position of each event described below.
 - A It will be hot tomorrow.
 - **B** The Sun will rise tomorrow.
 - **C** There will be floods in your town this year.
 - **D** There will be no car accidents in Sydney tomorrow.
 - E There are more than five people at your home at the same time today.
 - **F** Someone at school has a birthday today.
 - G It will rain tomorrow.
 - H The Sharks will win their next game.
 - I The next person to visit your classroom is male.
 - J Your favourite song comes on your radio station in the next hour.
- 7 Which term best describes the chance that the next baby born in Australia is a girl? Select the correct answer A, B, C, or D.

	A certain	B defi	nite	С	even chance	D	probable
8	Convert each number t	to a simp	plified fraction.				
	a 0.35	b 0.2	c	С	48%	d	6%
9	Evaluate each expression	on.	_				
	a $1 - \frac{1}{6}$		b $1 - \frac{3}{4}$		c 1	$-\frac{1}{1}$	$\frac{4}{0}$

11-01 Sample spaces

Probability is the branch of mathematics that studies the chances of different events occurring. In any situation, the set of all possible outcomes is called the **sample space**. For example, if a coin is tossed, the sample space is {heads, tails}. If each outcome has an equal chance, then we say that each outcome is **equally likely**.

Example

- a Write the sample space for this spinner.
- **b** How many outcomes are possible?
- c Is each outcome equally likely?

Solution

- a The sample space is {red, yellow, green, blue}.
- **b** There are 4 possible outcomes.
- **c** Each coloured region is equal in size $(\frac{1}{4}$ of the circle), so each outcome is equally likely.



The language of probability

This table lists some important probability terms, and how they apply to Example 1.

Probability term	In Example 1
An experiment is a situation involving chance	Spinning a spinner
that leads to results called outcomes.	
A trial is one go or run of the experiment.	One spin of the spinner
An outcome is the result of an experiment.	The arrow landing on green
The sample space is the set of all possible	{red, yellow, green, blue}
outcomes.	
An event is one or more outcomes of an	The arrow landing on a 'traffic light'
experiment.	colour: red, yellow or green
In a random experiment, every possible	All spins on this spinner are random
outcome has the same chance of occurring.	because every colour has the same
	chance

Example

- a How many outcomes are in the sample space when a die is rolled?
- **b** Is each outcome equally likely?

2

'Die' is the singular word for 'dice': one 'die', two or more 'dice'



Solution

- **a** The sample space is {1, 2, 3, 4, 5, 6}, so there are 6 possible outcomes.
- **b** Each outcome is equally likely, because each number has the same chance of coming up.

Example

A jar contains 5 red, 3 green, 6 yellow and 2 blue lollies. Taylor selects one lolly from the jar at random.

'At random' means that each lolly has an equal chance of being chosen

- **a** How many lollies are in the jar?
- **b** How many outcomes are in the sample space for the colour of the lolly?
- c Is each colour equally likely?

3

Solution

- **a** Number of lollies = 5 + 3 + 6 + 2 = 16
- **b** 4 outcomes {red, green, yellow, blue}
- **c** Each colour is not equally likely because there are different amounts of each colour. Some colours occur more frequently and have higher chances of being chosen.

Exercise 11-01 Sample spaces

1 For each spinner, write down the sample space and count the number of possible outcomes. See Example 1



- 2 For each experiment, count the number of possible outcomes and state whether each outcome See Example 2 is equally likely.
 - **a** tossing a coin
 - **b** the result of a soccer game when Australia plays South Korea
 - c the first letter of a person's name
 - **d** the gender of a baby
 - e the last digit of a phone number
 - **f** the result of a driving test
- 3 List the outcomes in each event.
 - a rolling an odd number on a die
 - $b \hspace{0.1in} \text{selecting a vowel from the letters of the alphabet}$
 - c having a house number greater than 4 but less than 10
 - d having a birthday in a month beginning with M
 - e living in a state capital city
 - **f** being in a primary school grade
- 4 This spinner is spun.
 - a How many outcomes are possible?
 - **b** Is each outcome equally likely? Explain your answer.
 - c Which outcome is most likely to occur?
 - d Which outcome is least likely?



- **5** A money box contains four \$2 coins, three \$1 coins, two 50c coins, six 20c coins and five 10c coins. It is shaken and one coin falls out at random.
 - a How many coins are in the money box?
 - **b** List the outcomes in the sample space for the type of coin.
 - **c** Is each type of coin equally likely?
 - d Which type of coin is most likely to fall out?
- 6 The 52 cards in a standard deck of playing cards are shown below, divided evenly into four suits: hearts, diamonds, clubs, spades.

Hearts:	¢ •	2	♥ ▲ :	}	¥ A A	\$	∜♥	¥ \$;	5 ₩ ♥ ▲ ▲ ▲\$	€¥ ▲	¥ A Aĝ						•	۲
Diamonds:	♠	2 •	 • • 	3	* * *	• •	4 ◆	 ◆ ◆ 	5	6.♦ ♦ ♦	 ♦ ♦ ∮ ∮ 	7		9 • • • • • • • • • • •		÷	•	
Clubs:	A * * *	*	*	3	* *	* ε	4 * *	* *	5*** ** ** **	6 * * *	* * *	7.* * * * * * * *	*** *** *** ***	⁹ .≁. ≁ .≁.≁ .*** *** *	¹⁰ ♣♣♣ ♣♣♣ ♥♥♥ ♥		÷	
Spades:	A • • •	2 *	 ▲ ✓ 	3	^ ♥ ♥	₽ ε	4 ♠ ♥	↑ ♥ [◆] _₽	5 * * * * * * *	6 ♠ ♥ ♥	♠ ♥ ♥9	₹ ↑ ↑ ♥ ♥ ♥ ♥ ♥ ♥	*** *** *** ***	9 • • • • • • • • • • • • • • • • • 6			÷.	

The cards are shuffled and one is taken out at random.

- **a** How many outcomes are in the sample space?
- **b** Is each outcome equally likely?
- c How many cards are in each suit?
- d How many red cards are in the deck?
- e How many 7s?
- f How many black Kings?
- g How many 8 of diamonds?
- **h** How many cards with an even number?
- i How many picture cards (J, K, Q)?
- j How many red Aces?
- 7 A 3-digit number is to be formed from the digits 1, 2, 5. Which of the following answers shows the sample space? Select A, B, C or D.
 - **A** {1, 2, 5} **B** {125}

 $\mathbf{C} \ \{125, 152, 215\} \qquad \mathbf{D} \ \{125, 152, 215, 251, 512, 521\}$

- 8 This spinner is spun.
 - a Write down the sample space of possible letters.
 - **b** Is each letter equally likely?
 - c Which letter is most likely?



- 9 Daniel is holding these 12 playing cards. Elvira picks a card without looking.
 - a How many 5s are held in the 12 cards?
 - **b** List the sample space of possible numbers.
 - c Which number is Elvira most likely to pick?
 - $d \ \ \, \text{List the sample space of possible colours.}$
 - e Which colour is Elvira more likely to pick?
 - f List the sample space of possible suits.
 - g Which suit is Elvira least likely to pick?

11-02 Probability

We can calculate the chance or probability of an event occurring as a fraction, percentage or a decimal.

Summary

P(E) means 'the probability of an event, E (occurring)'. If all possible outcomes are **equally likely**, then:

 $P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$ or $P(E) = \frac{\text{number of outcomes matching } E}{\text{number of outcomes in the sample space}}$

A **favourable outcome** is one of the outcomes in the event that you want, whose probability you are calculating.

Example 4

A die is rolled. Find the probability of rolling: **a** a 5 **b** a 7 **c** an odd number **d** a number less than 3 MAT07SPHS10035 Solution The sample space is $\{1, 2, 3, 4, 5, 6\}$. There are six possible outcomes and each outcome is equally likely. **a** $P(5) = \frac{1}{6}$ One chance in 6 MAT07SPPS00041 **b** $P(7) = \frac{0}{6} = 0$ No chance because we cannot roll a 7 on a die **c** $P(\text{odd}) = \frac{3}{6} = \frac{1}{2}$ 3 odd numbers out of 6: {1, 3, 5} **d** $P(\text{less than } 3) = \frac{2}{6} = \frac{1}{3}$ 2 numbers out of 6: {1 and 2}



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Example 🔪

TLF learning object The foul food maker (L215) A jar of lollies contains 5 red, 3 green, 6 yellow and 2 blue lollies. Taylor selects one lolly from the jar at random and notes its colour. Calculate each probability below.

- **a** P(yellow)
- **b** *P*(green or blue)
- c *P*(traffic light colour), as a percentage
- **d** P(not red), as a decimal

5

Solution

Total number of lollies in the jar = 5 + 3 + 6 + 2 = 16

a $P(\text{yellow}) = \frac{\text{Number of yellow lollies}}{\text{Total number of lollies}}$ $= \frac{6}{16}$ $= \frac{3}{8}$

b $P(\text{green or blue}) = \frac{\text{No. of green lollies} + \text{No. of blue lollies}}{\text{Total no. of lollies}}$ $= \frac{3+2}{16}$ $= \frac{5}{16}$

c P(traffic light colour) = P(red or green or yellow)

$$=\frac{5+3+6}{16} = \frac{14}{16}$$

Multiply by 100 to convert the fraction into a percentage.

$$\frac{14}{16} \times 100 = 87.5\%$$

=

=

d P(not red) = P(green or yellow or blue)

$$=\frac{3+6+2}{16} = \frac{11}{16}$$

= 0.6875 converting to a decimal



NEW CENTURY MATHS for the Australian Curriculum

Exercise 11-02 Probability

- 1 a List all the possible outcomes when a coin is tossed.
 - **b** How many different outcomes are possible?
 - c What is the probability of getting a tail?
 - **d** Write the probability of getting a tail as a percentage.
- **2** A die is rolled. Find the probability of rolling:
 - **a** a 4
 - **d** a number greater than 1
- **b** an even number**e** a prime number
- **c** a 10 **f** a number less than 7
- 3 For each spinner, calculate the probability of the pointer landing on red, as a percentage.



- 4 A money box contains four \$2 coins, three \$1 coins, two 50c coins, six 20c coins and five 10c coins. It is shaken and one coin falls out at random. Calculate each probability below.
 - a P(50c coin)

- **b** P(\$1 coin), as a decimal
- **d** P(not a 10c coin)

e P(gold coin), as a decimal

c P(10c or 20c coin), as a percentage

- **f** P(a coin under \$1), as a percentage
- **5** A packet of jellybeans has 4 yellow, 3 red, 6 green and 3 black jellybeans remaining. You tip the packet and one jellybean rolls out at random. What is the probability that it is not a red jellybean? Select the correct answer **A**, **B**, **C** or **D**.

A
$$\frac{4}{12}$$
 B $\frac{3}{16}$ **C** $\frac{3}{13}$ **D** $\frac{13}{16}$

6 A letter is selected at random from the alphabet. Find the probability that it is:

aMbRcM or Rda vowelenot a vowelfa letter before K in the alphabet

See Example 5



See Example 4

Extra questions Describing probability MAT07SPEQ00039

	7	 The needle on the spinner shown is spun. a Which colour has a probability of ¹/₃? b Which colour has a probability closest to ¹/₄? c Which colour has a probability of ¹/₆? d Which three colours have a combined probability of 50%?
orked solutions xercise 11-02 F07SPWS10060	8	A bag contains 6 white, 9 green and 5 blue marbles. Hannah takes one from the bag at random. Which colour has a probability of: a 0.25? b 45%? c $\frac{3}{10}$?
	9	 A computer generates a random number from 1 to 10. Find the following probabilities. a P(a square number) b P(a number less than 4), as a decimal c P(a prime number), as a percentage d P(a factor of 10) e P(a number greater than 6), as a decimal f P(a 5 or 7), as a percentage
	10	What is the probability that a baby is born on a weekday rather than a weekend, if each day is equally likely?
	11	Draw and label a spinner that would produce the following probabilities: $P(\text{red}) = \frac{1}{9}$, $P(\text{yellow}) = 24\%$, $P(\text{blue}) = \frac{1}{2}$, $P(\text{black}) = 20\%$.
	12	Ryan bought 6 raffle tickets at the school fete. If a total of 800 tickets was sold, what is the probability that Ryan wins first prize? Express your answer:
	13	a as a fraction b as a decimal Your maths teacher calls out a name randomly from your class roll. What is the probability that it is:
		a your name?b a girl's name?c someone aged 12?d someone with blond hair?
	14	A deck of 52 playing cards is shuffled and one is taken out at random. Find the probabilitythat it is:a a red cardb a club cardc the King of spadesd a Queene a card with an odd numberf a black picture cardg a red 7h a 4
	15	You may refer to the diagram from question 6 , Exercise 11-01 , on page 444. An Esky contains 8 cans of lemonade, 5 cans of orange drink and 2 cans of lime drink. How many cans of cola must be added so that the probability of randomly selecting: a a can of lemonade is 50%? b an orange drink is 0.25? c a lime drink is ¹ / ₂
	16	What is the probability that a person randomly chosen has a birthday in a month beginning with the letter J? Select the correct answer A, B, C or D. A $\frac{1}{12}$ B $\frac{1}{6}$ C $\frac{1}{4}$ D $\frac{1}{3}$

17 There are 16 teams in a football competition. Paula, therefore, believes that her favourite team, The Bulldogs, has a probability of $\frac{1}{16}$ of winning the competition. Discuss with your friends whether Paula is correct and write your answer, giving reasons.

MA

- 18 A student council is made up of a number of boys and girls. One member of council is selected at random to attend the regional conference. How many boys and girls could there be in the council if the probability of selecting a boy is $\frac{5}{9}$?
- **19** A fish tank has 8 red and 14 yellow fish. One fish is randomly selected from the tank.
 - **a** Write the probability that the fish is yellow as a decimal correct to 2 decimal places.
 - **b** Write the probability that the fish is red as a percentage.
 - **c** How many red fish must be removed from the tank to make the probability of selecting a red fish 0.3?



- 20 a What is the lowest probability value? Name an event that could have this value.
 - **b** What is the highest probability value? Name an event that could have this value.

Just for the record

Insurance

Insurance is a financial scheme that offers money in times of loss or damage. People can insure their houses against fire, flood or theft. You pay an amount each month or year, and if the loss or damage happens, the insurance company pays you a large sum of money to replace the insured item.

Insurance was invented in ancient Babylon around 3000 BCE, when voyagers and traders insured their ships against damage or loss. Modern mathematicians working for insurance companies use the statistics of past losses and deaths to predict the chances of future losses and deaths. They have found some interesting probabilities, such as those shown in the table.



Event	Chance
Being in a house fire	1 chance in 800
Going to prison	Male: 1 chance in 800
	Female: 1 chance in 24 000
Dying in a traffic accident	1 in 5000
Dying at age 5	1 chance in 10 000
Being struck by lightning	1 chance in 280 000
Dying in a plane crash	1 chance in 11 million

An actuary works in the insurance industry. Find out what an actuary does.

11-03 The range of probability



Because probability is a fraction

 $P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$

its value must range from 0 to 1, or as a percentage, from 0% to 100%.



- The probability of an **impossible** event is 0.
- The probability of a **certain** event is 1.
- The probability of any event ranges from 0 to 1.

A probability close to 1 indicates an event that is very **likely** or probable. A probability close to zero indicates an event that is very **unlikely** or improbable.

Example 6

Five cards numbered 11, 5, 9, 7 and 3 are shuffled and Jo chooses one at random. Find the probability that the number chosen is:

a divisible by 3 b less than 12 c a factor of 45 d even

Solution

-		
a	$P(\text{divisible by } 3) = \frac{2}{5}$	2 numbers {9, 3}
b	$P(\text{less than } 12) = \frac{5}{5} = 1$	All 5 numbers are less than 12: a certain event.
с	$P(a \text{ factor of } 45) = \frac{3}{5}$	3 numbers {5, 9, 3}
d	$P(\text{even}) = \frac{0}{5} = 0$	None are even: an impossible event

Exercise 11-03 The range of probability 1 If a six-sided die is rolled, find the probability of rolling: See Example 6 **c** a factor of 12 **a** a 5 **b** an 8 **d** a prime number **e** a number less than 7 **f** a multiple of 3 **2** One letter is selected at random from the word EVACUATION. Find the percentage probability that it is: c X a V **b** A **d** a vowel 3 Daniel is holding these 12 playing cards. Elvira picks a card without looking. Find the probability that it: **a** is red **b** shows an even number **c** is a picture card **d** has the number 7 e is a hearts card **f** not an Ace 4 A basketball team captain is to be chosen randomly from four candidates, Carl, Lee, Su and Aldo. What is the probability that the captain chosen is: a Aldo? b Su? c Manjeet? 5 Match each probability value to its correct description: $\frac{1}{2}$ **b** 0 c 90% **d** 1 a $\frac{3}{4}$ **f** 0.1 **h** 2% **g** 0.6 e A cannot happen **B** better than average chance **C** even chance **D** good chance E very likely **F** almost impossible G slim chance H must happen 6 A coin falls out of a piggy bank containing four 5-cent, five 10-cent, six 20-cent and three \$1 coins. Find the probability that it is: a a 5c coin **b** a \$1 coin **c** a 50c coin f not a 20c coin **d** a 10c or 20c coin e not a \$1 coin 7 A traffic light shows red for 63 seconds, green for 99 seconds and yellow for 2 seconds. Find as a decimal the probability that it shows: a green **b** yellow **c** red or green **d** not red, yellow or green 8 A computer generates a random number from 1 to 20. Find as a percentage the probability that the number is: **a** a multiple of 4 **b** a factor of 20 **c** a number less than 30 **e** a prime number f divisible by 5 d 25

9 A die has 2 faces blue, 1 face red and the other faces green. It is rolled once. Match each event to its correct probability: a red **b** blue **c** yellow **d** a colour that is not red **e** blue, red or green **f** a traffic light colour **B** 50% **F** $\frac{1}{6}$ **C** $\frac{5}{6}$ **D** $\frac{1}{3}$ A **G** 100% **E** 0 10 A letter is chosen at random from the words 'NEW CENTURY MATHS'. Find the following Worked solutions probabilities. Exercise 11-03 **b** $P(\mathbf{M})$ **a** P(T)P(N or E)MAT07SPWS10061 e *P*(a letter also found in PROBABILITY) **d** P(a consonant) $\mathbf{f} P(\mathbf{K})$ 11 Write an event that could have a probability of: $\frac{1}{2}$ a **b** 0 **c** 90% **d** 1 $e \frac{\overline{3}}{4}$ **f** 0.1 **g** 0.6 **h** 2%

Investigation: The Greedy Pig game

Puzzle sheet	This is a gan
Greedy Pig game	die and a sco
MAT07SPPS10041	1 All playe
Weblink	2 Before th
The Monty Hall	option of
problem	remainin
	of increa

This is a game for four or more players. You need a playing die and a score sheet (use the link to print one out).

- 1 All players stand at the start of a round. A die is rolled twice and the total is each player's starting score.
- 2 Before the next roll of the die, each player has the option of sitting down and keeping their score, or remaining standing and 'being greedy' with the chance of increasing their score from the next roll of the die.
- 3 The die is rolled and the number is added to the score of each player standing, but if 2 comes up, all standing players lose their points.
- 4 A round ends when a 2 is rolled or all players are seated. Each player then records their score for the round.
- **5** The round is repeated four more times and the overall winner is the player with the highest total score for the five rounds.
- 6 Play this game several times. Is there any strategy to winning?

Just for the record

Buckley's chance

When someone says you have 'Buckley's chance' of something happening, it means that you have little or no chance at all. This is old Australian slang based on the name of a Melbourne department store in the 1850s called 'Buckley and Nunn'. The original expression was 'You have two chances: Buckley's and none'.

English is often a funny and illogical language. For example, 'slim chance' and 'fat chance' sound like opposites but they both mean the same thing.

Find another slang expression that describes a chance.



1-04 Experimental probability

Probability calculated using the formula:

 $P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$

is more specifically called the theoretical probability (or calculated probability). We can also determine probability based on the results of an experiment or trial that has been repeated many times, such as testing the safety of different cars, or rely on past statistics, such as the number of rainy days in April. This type of probability is called **experimental probability** or relative frequency, and has the following formula.

SummaryProbability 2
$$P(E) = \frac{\text{number of times the event happened}}{\text{total number of trials}}$$
MATO7SPHS10036or $P(E) = \frac{\text{frequency of } E}{\text{total frequency}}$ Frequency means the number of times something happens

Example

7

Selina spun this spinner 80 times and found the following results:

Outcome	Red	Green	Yellow	
No. of times	44	25	11	

- **a** What is the theoretical probability of spinning red?
- **b** For 80 spins, what is the expected number of times of spinning red? How does this compare with the actual number of times?
- **c** What is the experimental probability of spinning red?

Solution

- **a** $P(\text{red}) = \frac{5}{10} = \frac{1}{2}$ **b** Expected number of reds $= \frac{1}{2} \times 80 = 40$.

From the table, the observed number of reds = 44, which is close to the expected number.

c Experimental $P(\text{red}) = \frac{44}{80} = \frac{11}{20}$



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MAT07SPPS00039

Summary

The expected number of times an event will occur over repeated trials is called the **expected frequency**.

Expected frequency = theoretical probability \times number of trials

Exercise 11-04 Experimental probability

See Example 7

Extra questions

Experimental

probability

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Worked solutions

Exercise 11-04 MAT07SPWS10062

MAT07SPPS00038

MAT07SPWK00062

MAT07SPWK00063

- 1 For the spinner in Example 7, find:
 - a the theoretical probability of spinning yellow
 - **b** the expected number of times of spinning yellow and how this compares with the observed number of times
 - c the experimental probability of spinning yellow.
- 2 A pair of dice was rolled 50 times and their sum calculated each time. The results are shown in this table.

Sum	2	3	4	5	6	7	8	9	10	11	12
Frequency	0	2	4	6	5	5	9	6	8	3	2

a Find, as a decimal, the experimental probability of rolling a sum:

i of 10	ii of 7 or 11	iii that is even
iv less than 6	v greater than 10	vi that is a prime number

- **b** Which sum:
 - i was most likely?

iv had a probability of 10%?

- ii had a probability of $\frac{2}{25}$? v was second-most likely?
- iii was least likely? vi had a probability of $\frac{4}{25}$?

a Copy this table.

3

Outcome	Tally	Frequency
Head		
Tail		
	Total	

- **b** Toss a coin 50 times and record the result for each toss in the table.
- c What is the theoretical probability of tossing a tail?
- **d** In 50 tosses, what is the expected frequency of tails? How does this compare with the actual frequency?
- e What is the experimental probability of tossing a tail?
- **f** What do you think would happen to the experimental probability of a tail if the coin was tossed 1000 times rather than 50?



4 The exact amounts of soft drink in 50 cans of drink were measured and the results are shown below.

Amount (mL)	373	374	375	376	377
Number of cans	2	6	38	3	1

Using these results, if you bought a can of drink, what is the probability that it will contain exactly 375 mL of drink? Select the correct answer **A**, **B**, **C** or **D**.

A $\frac{375}{50}$	B $\frac{19}{25}$	C $\frac{6}{25}$	D $\frac{3}{50}$

5 a Copy this table.

Outcome	Tally	Frequency
1		
2		
3		
4		
5		
6		
	Total	

- **b** Roll *a* die 50 times and record the result for each roll in the table.
- **c** Find the theoretical probability and expected frequency of rolling:

i	3	ii	6
iii	an even number	iv	a number below 6

d Find the experimental probability of rolling:

i3ii6iiian even numberiva number below 6

- e How do the observed frequencies compare with the expected frequencies?
- ${f f}$ What would you expect to happen if the die was rolled 1000 times?
- 6 A die was rolled 80 times, with the results shown below.

Outcome	1	2	3	4	5	6
Frequency	11	13	9	13	12	22

Worked solutions Exercise 11-04 MAT07SPWS10062

- a Is each outcome equally likely?
- **b** Do you think this die is loaded (unfair)? Give a reason for your answer.
- c Write the experimental probability of rolling a 1 on this die, as a percentage.
- d If this die was rolled 100 times, how many times would you expect 3 to come up?
- 7 Tamara tossed a coin many times and got 140 heads and 110 tails. Calculate the experimental probability of heads with this coin as a percentage.







Technology Rolling a die

Simulations can be used to model or replicate real-life events. Mathematicians, actuaries, scientists and statisticians can often simulate events on a computer, which saves time and money because large amounts of data can be obtained in a relatively short period of time.

In this activity, a graphics calculator is used to simulate the rolling of a normal die with sides 1, 2, 3, 4, 5 and 6.

Note: This activity uses a Casio graphics calculator.

1 Copy the table shown below.

Outcome	Number of times rolled
1	
2	
3	
4	
5	
6	

Technology worksheet
Excel Cointossing experiment
MAT07SPC100022
TLF learning object
Dice duels (L2641)
TLF learning object
Dice duels: tool (L2645)

2 Using the RUN mode, enter the following formula: Int(Ran# \times 6 + 1) as shown below.



- 3 Repeat the simulation 20 times and record the results in your table.
- 4 Are certain numbers more likely to be rolled than others? (For example is a 2 more likely to be rolled than a 5?) Do your results reflect this?
- 5 Compare your results with the simulated results of your class. Are they similar or different? Are they what you and your classmates expected? Discuss.

Mental skills 11 Maths without calculators

Adding 90c and 95c

A quick way to mentally add monetary amounts ending in 90c or 95c is to round up to whole dollars and then subtract 10c or 5c respectively.

1 Study each example.

a
$$\$8.95 + \$17.95 + \$11.95 = \$9 + \$18 + \$12 - 5c - 5c - 5c$$

 $= \$39 - 15c$
 $= \$38.85$
b $\$14.90 + \$5.95 + \$12.90 = \$15 + \$6 + \$13 - 10c - 5c - 10c$
 $= \$34 - 25c$
 $= \$33.75$
c $\$18.90 + \$24.90 + \$8.80 = \$19 + \$25 + \$9 - 10c - 10c - 20c$
 $= \$53 - 40c$
 $= \$52.60$

- 2 Now simplify each expression.
 - **a** \$12.95 + \$7.95 + \$14.95
 - **c** \$9.95 + \$11.90 + \$13.90
 - **e** \$13.95 + \$19.95 + \$8.95
 - **g** \$27.95 + \$16.95 + \$12.90
 - i \$24.95 + \$34.95

- **b** \$20.90 + \$4.95 + \$13.95
- **d** \$11.95 + \$15.90 + \$6.95
- **f** \$34.90 + \$34.90
- **h** \$11.90 + \$21.95 + \$19.80
- **j** \$33.90 + \$49.90 + \$49.90

Investigation: Complementary events

In everyday life, we use the word 'complementary' to describe things that go together and 'complete the picture' when they are together. For example, when dressing for an occasion:

- a shirt and a matching tie complement each other
- a dress and a matching pair of shoes complement each other

Remember also that 'complementary' angles add to 90°.

In probability, **complementary events** are events that together make up all the possible outcomes.

Event	Complementary event
Tossing a tail on a coin	Tossing a head on a coin
Rolling a 6 on a die	Rolling any of the other numbers, from 1 to 5, on a die
Raining	Not raining
Being born on a Monday	Being born on a day other than Monday

- 1 Suppose that there is an equal chance of being born on any day of the week: Monday to Sunday.
 - **a** What is *P*(Tues), the probability of being born on a Tuesday?
 - **b** What is *P*(not Tues), the probability of being born on a day other than Tuesday?
 - c What do you notice about P(Tues) + P(not Tues)?
- **2** A fruit bowl contains 7 apples, 4 oranges and 9 bananas. One piece of fruit is selected at random from the bowl.
 - a Find *P*(orange) b Find *P*(not orange)
 - **c** What do you notice about P(orange) + P(not orange)?
- 3 A baby is selected at random from the maternity section of a large hospital. There is an equal chance of the baby being a boy or a girl.
 - **a** Find *P*(boy) **b** Find *P*(girl)
 - **c** What do you notice about P(boy) + P(girl)?
- 4 Copy and complete the following sentence:

The probability of an event _____ the probability of its complementary event must always equal _____.

11-05 Complementary events

In any situation, the probabilities of all possible outcomes must add to 1.

Complementary events are events that together make up all the possible outcomes, such as a head and a tail when tossing a coin. The **complement** of an event *E* means all of those outcomes that are *not E*, or that are the 'opposite' of *E*.

Because an event and its complement covers all possible outcomes, the sum of their probabilities must equal 1.

Summary

$$\begin{split} P(E) + P(\text{not } E) &= 1\\ \text{or } P(\text{not } E) &= 1 - P(E)\\ \text{or } P(\text{complementary event}) &= 1 - P(\text{event})\\ \text{or } P(\text{event not occurring}) &= 1 - P(\text{event occurring}) \end{split}$$

If the probability is written in percentage form, then P(not E) = 100% - P(E).

Example 8

On this spinner, what is the probability of spinning:

- a red?
- **b** a colour that is not red?
- **c** a colour that is not green?

Solution

a
$$P(\text{red}) = \frac{2}{6} = \frac{1}{3}$$

b $P(\text{not red}) = 1 - P(\text{red})$
 $= 1 - \frac{1}{3}$

$$=\frac{2}{3}$$

Note that $P(\text{red}) + P(\text{not red}) = \frac{1}{3} + \frac{2}{3} = 1$, which covers all possible outcomes.

c
$$P(\text{green}) = \frac{1}{6}$$

 $P(\text{not green}) = 1 - P(\text{green})$
 $= 1 - \frac{1}{6}$
 $= \frac{5}{6}$
Note that $P(\text{green}) + P(\text{not green}) = \frac{1}{6} + \frac{5}{6} = 1$,
which covers all possible outcomes.



red

white

green

white

white

red

YEAR 8



MAT07SPWK00064

		Exercise 11-	05 Compleme	entary events
	1	Write the complement a The sex of a new c Losing a soccer m e Being over 18 yea g A traffic light show	nt for each event. baby being female hatch rs of age wing green	 b Rain tomorrow d Being left-handed f Choosing a heart from a deck of cards h A train arriving late
See Example 8	2	A die is rolled. What a 3? d not a 3?	is the probability thatb less than 3?e not less than 3?	at the result is: c prime? f not prime?
	3	Kylie buys a ticket in the probability of Ky a winning the prize: A jar contains jellybe red 40, blue 25, black What is the probabil a white?	a raffle in which 1000 die: eans in the following co k 50, white 15. One jel ity that the jellybean is b not white?	 b not winning the prize? colours and amounts: ellybean is selected at random. is: c yellow? f not blue or black?
	5	 Write the probability a The probability of b The chance of sho c The probability or 	of the event that is co f choosing a Jack from poting a basketball hoc f winning a prize is 0.0	complementary to each of the following events. In a pack of cards is $\frac{1}{13}$. Soop is 55%.
Worked solutions Exercise 11-05 MAT07SPWS10063	6	In a bag of toy cars the random, the chance of a What is the chance b What is the chance c If the bag holds 3 d What is the chance	here are only three col of it being red is 0.5, a ce of selecting 'red or w ce of the car you select 0 cars, how many of ea ce of the car you select	blours: red, blue and white. If you take out a car at and the chance of it being white is 0.2. white'? et being blue? each colour would you expect to find? et being pink?
	7	 What is the probabil a beginning with the Four students, Sue, I bag. A card is chosen a Find the probabil b Find the probabil c What is the chance d What is the chance 	ity of the next person y e letter A? Liam, Emily and Matt, n, without looking, to s ity that Emily was not ity that the captain is a ce that the captain is no ce the captain is the tea	 b that does not begin with the letter A? t, write their names on cards and place the cards in a select the class captain. t chosen. a boy. not a boy? eacher?
	9	Which of the followi answer A , B , C , or D A coming last C not winning the ra	ng is the complementa). ace	tary event to 'winning a race'? Select the correctB coming second or thirdD coming second

- 10 In a football match, the Eels have a 43% chance of winning, while the Tigers have a 49% chance of winning.
 - **a** What other outcome is possible? **b** What is the probability of this outcome?
- 11 The probability that a man has skin cancer is 0.03. What is the probability that a man doesn't have skin cancer? Select the correct answer A, B, C or D.
 - A 0.2 **B** 0.7 C 0.07 **D** 0.97

12 The letters of the word PROBABILITY are written on separate cards and one is randomly selected. What is the probability that a letter drawn out is:

- a not P? **b** not a vowel? c not I? d not A or B?
- 13 The probability that it will rain this weekend is 85%. What is the probability that it won't rain?
- 14 What is the decimal probability that a mobile number selected at random doesn't end in 0 or 1?

Exercise 11-05 MAT07SPWS10063

Worked solutions

Power plus

- Two dice are rolled and the sum of the numbers is calculated. 1
 - Copy and complete this table to show all possible sums a
 - **b** How many different sums are possible?
 - c Why isn't each sum equally likely?
 - **d** Which sum is most likely?
 - e Which sum is least likely?
 - What is the probability of a sum of 2? f
 - What is the probability of a sum of 10? g
 - h Which sum has a probability of $\frac{1}{9}$?
- Three coins are tossed together. One possible outcome is HHT, that is, heads on the first 2 coin, heads on the second coin, tails on the third coin.
 - How many other possible outcomes are there? a List them.
 - **b** Use your answers from part **a** to calculate the probability of:
 - **i** three heads ii two heads
- iii one head
- iv no heads **v** at least one head
- vi at most one head



- List all possible meal deal combinations. a How many are there?
- **b** If you were to choose a combination at random, what is the probability that it will:
 - be pizza, lemonade and cake? i
 - not contain a hot dog? ii
 - iii contain cola?



e sums.							
			First die				
	+	1	2	3	4	5	6
	1	2	3	4	5	6	7
e	2	3	4				
d di	3	4					
con	4						
s	5						
	6						

Chapter 11 review

Language of maths

MAT07SPPS10043

Quiz	
Probability	

MAT07SPQZ00009

MAT07SPWK00065

certain chance complementary event die/dice equally likely event

expected frequency experiment favourable frequency impossible

improbable likely experimental probability observed outcome possible probability

- probable random sample space theoretical probability trial unlikely
- 1 What is the opposite of **impossible** and what is its probability value?

2 What is the difference between impossible and improbable?

- 3 What is the name given to the set of all possible outcomes for a situation?
- 4 What is the **complementary event** to a baby being born on a weekend?
- 5 What does it mean when a name is drawn out of a box 'at random'?
- What word means the number of times something happens? 6

Topic overview

MAT07SPWK10093

Write about what you have learnt in this chapter. Write down the parts of the chapter that were new to you. • Copy and complete: The things I understand about probability are... The things I am still not confident about in this chapter are... Print (or copy) and complete this mind map of

the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Chapter 11 revision

1	List the outcomes for the sample space in each situation. a The result of a driving test b The classification of a new film (for example, PG)	See Exercise 11-01
	c The winner of a tennis match when Katrina plays Biljanad The type of public transport that goes to your school	
2	 a For this spinner, how many outcomes are there in the sample space? b Why isn't each outcome equally likely? c Which number is most likely? d Which number is least likely to be spun? e Which number has a probability of 25%? 	<i>See</i> Exercise 11-01
3	A die is rolled. What is the probability of rolling:	See Exercise 11-02
	a a 1? b an odd number? c a number less than 5?	
4	A truck carries 325 boxes of exercise books, 210 boxes of rulers, 360 boxes of paper and 145 boxes of pens. If a box is taken at random from the truck, what is the probability (as a percentage) that the box contains:	<i>See</i> Exercise 11-02
	a rulers? b no pens?	
_	c exercise books or paper? d erasers?	0 5 : 11.07
)	A jar contains 1 red, 6 yellow, 2 white and 3 black jellybeans. If a jellybean is selected at random, find the probability that it is:	See Exercise 11-03
	a yellow b white or red c blue d not green	
6	Which word best describes an event that has a probability of 0.29? Select the correct answer A, B, C, or D.	See Exercise 11-03
	A unlikelyB highly probableC almost impossibleD good chance	
7	Which event below could be described as a certain event? Select the correct answer A, B, C, or D.	See Exercise 11-03
	A choosing a 4 from a hat containing the numbers 1, 2, 3 and 4B choosing a red ball from a bag of green balls	
	C rolling an odd or even number on a die D a new baby being a girl	
8	A coin was tossed 80 times and landed heads 24 times.	See Exercise 11-04
-	a What is the theoretical probability of obtaining a head?	
	b What is the expected frequency of heads from 80 tosses of a coin? How does the observed frequency compare to this?	
	c What is the experimental probability of obtaining a head?	
9	Write the complementary event for each of the following events and its probability.	See Exercise 11-05
	a Choosing a queen from a standard deck of cards.b Rolling a multiple of 3 on a die.	
	e Buying the winning ticket out of 550 tickets sold.	
0	There are 8 brown marbles, 4 black marbles and 3 white marbles in a bag. One marble is	See Exercise 11-05