3

Number and algebra

Whole numbers

The ability to use numbers is an essential part of modern living. Think of all the times during the day when you use numbers – buying food, catching buses and trains and even in sport.

Chapter outline

	Proficiency strands			
3-01 Rounding and estimating	U	F		
3-02 Multiplying numbers	U			
3-03 Dividing numbers	U			
3-04 Divisibility tests	U	F	PS	R
3-05 Long division	U			
3-06 Roman numerals	U			
3-07 Powers and index notation	U			
3-08 Square root and cube root	U	F		R
3-09 Prime and composite numbers	U	F		R
3-10 Prime factors	U			
3-11 Greatest common divisor	U	F	PS	R
3-12 Lowest common multiple	U	F	PS	R

Wordbank

composite number A number with more than two factors

divisibility test A rule for testing whether or not a number is divisible by a specific value, for example, divisible by 3

factor A value that divides evenly into a given number, for example, 3 is a factor of 15

factor tree A diagram that lists the prime factors of a number

greatest common divisor (GCD) The largest factor shared by two or more numbers, also called the **highest common factor (HCF)**

index notation A way of writing powers for the repeated multiplication of a number, for example, 3^5

prime number A number with only two factors, 1 and itself

square root (of a number, symbol $\sqrt{\ }$) The positive value which, if squared, gives the number

In this chapter you will:

Maths clip
Whole numbers
MAT07NAVT00002

- use estimation and rounding to check the reasonableness of answers to calculations
- select and apply efficient mental strategies and appropriate digital technologies to solve problems involving multiplying and dividing whole numbers
- use simple divisibility tests
- understand that if a number is divisible by a composite number then it is also divisible by the factors of that number
- divide by a two-digit number
- recognise, read and convert Roman numerals
- investigate index notation
- investigate and use square roots of perfect square numbers, cube roots of perfect cube numbers
- define and compare prime and composite numbers and explain the difference between them
- represent whole numbers as products of powers of prime numbers
- solve problems involving lowest common multiples and greatest common divisors (highest common factors) for pairs of whole numbers by comparing their prime factorisation

SkillCheck

Worksheet
StartUp assignment 3
MAT07NAWK10018

Worksheet
Calculation aids
MAT07NAWK10019

Worksheet
Number grids
MAT07NAWK10020

Puzzle sheet
Cross number puzzle
MAT07NAPS10008

Puzzle sheet
Magic squares
MAT07NAPS10009

1 Is 691 odd or even? How can you tell?

2 Is 270 divisible by 5? How can you tell?

3 Evaluate each product.

a 7×3 b 6×5 c 4×8 d 3×6
e 9×4 f 5×8 g 7×7 h 6×9
i 8×8 j 9×7 k 4×6 l 7×8
m $-3 \times (-3)$ n 9×9 o 5×5 p $4 \times (-4)$

4 Write all the factors of:

a 20 b 12 c 19

5 Is 166 divisible by 2? How can you tell?

6 Evaluate each quotient.

a $24 \div 3$ b $35 \div 7$ c $24 \div 4$ d $36 \div 6$
e $20 \div 5$ f $72 \div 9$ g $48 \div 8$ h $32 \div 4$
i $28 \div 7$ j $45 \div 5$ k $54 \div 9$ l $63 \div 7$

7 Write the first 5 multiples of:

a 8 b 7 c 12

8 Is 34 divisible by 10? How can you tell?

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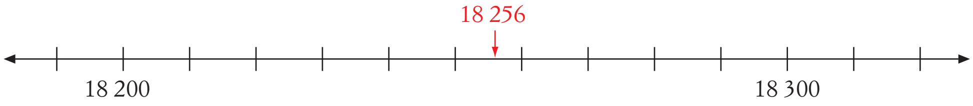
3-01 Rounding and estimating

Rounding numbers

When calculating with numbers it is often useful to have a rough idea of the answer before actually working it out. Rounded numbers are easier to work with and to compare. For example, a town's population of 18 256 can be rounded to 18 300 (to the nearest hundred) because 18 256 is closer to 18 300 than 18 200.



Skillsheet
Place value
MAT07NASS10011

Skillsheet
Reading and writing large numbers
MAT07NASS10012

Skillsheet
Rounding whole numbers
MAT07NASS10013

Homework sheet
Whole numbers
MAT07NAHS10001

Summary

To **round a number**, 'cut' it at the required place and look at the digit in the next place:

- if the digit is less than 5 (that is 0, 1, 2, 3 or 4), **round down**
- if the digit is 5 or more (that is 5, 6, 7, 8 or 9), **round up**

Example 1

Round 8470 to the nearest hundred.

Solution

Counting by hundreds, 8470 is between 8400 and 8500.

- In 8470, the hundreds digit is **4**
- The digit in the next (tens) place, 7, is more than 5, so **round up** to 8500.

8470 ≈ 8500 (rounded to the nearest hundred).

The symbol '≈' means 'approximately equal to'.

Example 2

Round 247 182 to the nearest thousand.

Solution

In 247 182, the thousands digit is 7 and the next digit is 1.
1 < 5, so round down to 247 000. The thousands digit, 7, stays the same.

247 182 ≈ 247 000 (rounded to the nearest thousand).

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Estimating answers

A quick way of estimating an answer is to round each number in the calculation.

Example 3

Estimate the answer to each expression.

a $631 + 280 + 51 + 43 + 96$ **b** 67×12
c $55 + 132 - 34 + 17 - 78$ **d** $510 \div 24$

Solution

a $631 + 280 + 51 + 43 + 96$
$\approx 600 + 300 + 50 + 40 + 100$ Estimating
$= (600 + 300 + 100) + (50 + 40)$
$= 1000 + 90$
$= 1090$ (Exact answer = 1101)

b $67 \times 12 \approx 70 \times 10$ Estimating
$= 700$ (Exact answer = 804)

c $55 + 132 - 34 + 17 - 78$
$\approx 60 + 130 - 30 + 20 - 80$ Estimating
$= (60 + 20 - 80) + (130 - 30)$
$= 0 + 100$
$= 100$ (Exact answer = 92)

d $510 \div 24 \approx 500 \div 20$ Estimating
$= 50 \div 2$
$= 25$ (Exact answer = 21.25)

Exercise 3-01 Rounding and estimating

See Example 1
1 Round each number to the nearest hundred.
a 3148 **b** 49 028 **c** 2597 **d** 4 934 277

See Example 2
2 Round each number to the nearest thousand.
a 23 538 **b** 45 370 **c** 62 941 **d** 47 929

3 Round each number to the nearest ten.
a 45 819 **b** 1699 **c** 8314 **d** 71 262

4 Round 64 218 to the nearest:
a thousand **b** hundred **c** ten **d** ten thousand

5 Round 1 327 509 to the nearest:
a thousand **b** ten thousand **c** ten **d** million

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6 Estimate the answer for each expression.

See Example 3

a $27 + 11 + 87 + 142 + 64$
b $55 + 34 - 22 - 46 + 136$
c $684 + 903$
d $35 + 81 + 110 + 22 + 7$
e $517 - 96$
f $210 - 38 - 71 + 151 - 49$
g $766 - 353$
h 367×2
i 83×81
j 984×16
k $828 \div 3$
l $507 \div 7$

7 Over the holidays, 27 792 people visited a museum. Write this figure correct to the nearest hundred.

8 The extensions on Nina's house are quoted as costing $17 464. Write this amount correct to the nearest $100.

9 Write a number that can be rounded to:

a 370 b 5400 c 12 900 d 6000

10 The crowd at a football Grand Final was 104 427. Round this figure to the nearest thousand.

11 The distance between Sydney and Brisbane is 998 km. Round this distance to the nearest:

a 10 km b 100 km c 1000 km

12 The population of Australia is 23 581 800. Round this figure to the nearest thousand.

Worked solutions
Exercise 3-01
MAT07NAWS10012

3-02 Multiplying numbers

Worksheet
Whole numbers 6
MAT07NAWK00022

Worksheet
Calculation aids
MAT07NAWK10019

Worksheet
Number grids
MAT07NAWK10020

Puzzle sheet
Cross number puzzle
MAT07NAPS10008

Puzzle sheet
Magic squares
MAT07NAPS10009

Example 4

Evaluate each product.

a 243×6 b 573×36

Solution

a
```
  243
×   6
 1458
```
Check by estimating: $243 \times 6 \approx 200 \times 6 = 1200$

b
```
   573
×   36
  3 438
 17 190
 20 628
```
Check by estimating: $573 \times 36 \approx 600 \times 40 = 24\ 000$

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TLF learning object
Rectangle multiplication (L3503)

Weblink
Finger multiplication

Puzzle sheet
Find the quote 2

MAT07NAPS00013

Mental multiplication

Multiplying by:	Mental strategy
2, 4 or 8	Double once, two times or three times respectively
5	Halve, then multiply by 10 (because $\frac{1}{2} \times 10 = 5$)
9	Multiply by 10, then subtract the number
10	Insert 0 at the end of the number
100	Insert 00 at the end of the number

Example 5

Evaluate each product.

a 68×4 **b** 36×5 **c** 12×9 **d** 14×8

Solution

a $68 \times 4 = 68 \times 2 \times 2$
$= 136 \times 2$
$= 272$

Double twice
$68 \times 2 = 60 \times 2 + 8 \times 2 = 120 + 16 = 136$
$136 \times 2 = 130 \times 2 + 6 \times 2 = 260 + 12 = 272$
Estimate: $68 \times 4 \approx 70 \times 4 = 280$

b $36 \times 5 = 36 \times \frac{1}{2} \times 10$
$= 18 \times 10$
$= 180$

Because $\frac{1}{2} \times 10 = 5$
Insert a 0 at the end
Estimate: $36 \times 5 \approx 40 \times 5 = 200$

c $12 \times 9 = 12 \times 10 - 12$
$= 120 - 12$
$= 108$

Multiply by 10, then subtract the number
Estimate: $12 \times 9 \approx 12 \times 10 = 120$

d $14 \times 8 = 14 \times 2 \times 2 \times 2$
$= 28 \times 2 \times 2$
$= 56 \times 2$
$= 112$

Double 3 times
$14 \times 2 = 10 \times 2 + 4 \times 2 = 20 + 8 = 28$
$28 \times 2 = 20 \times 2 + 8 \times 2 = 40 + 16 = 56$
$56 \times 2 = 50 \times 2 + 6 \times 2 = 100 + 12 = 112$
Estimate: $14 \times 8 \approx 14 \times 10 = 140$

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Exercise 3-02 Multiplying numbers

1 Copy and complete each multiplication. See Example 4

a $\begin{array}{r} 34 \\ \times\ 7 \\ \hline \end{array}$ b $\begin{array}{r} 219 \\ \times\ \ 5 \\ \hline \end{array}$ c $\begin{array}{r} 28 \\ \times 16 \\ \hline \end{array}$ d $\begin{array}{r} 325 \\ \times\ 21 \\ \hline \end{array}$

2 Lara loves to play tennis. She pays $7 each time she plays. How much does Lara pay to play 42 times a year?

3 Evaluate each product.

a 14×8 b 238×3 c 344×9 d 506×7
e 1084×3 f 45×14 g 64×25 h 107×32

4 A bus route is 46 kilometres long. A bus makes 12 trips in one day. Find the total distance travelled each day.

5 Nathan can type 76 words per minute. How many words can he type in 15 minutes?

6 Evaluate each product mentally. See Example 5

a 85×2 b 39×2 c 64×2 d 57×4
e 28×4 f 44×4 g 16×8 h 33×8

7 Evaluate each product mentally.

a 22×5 b 14×5 c 28×5 d 36×5
e 54×5 f 82×5 g 16×5 h 48×5

8 Evaluate each product mentally.

a 34×9 b 51×9 c 27×9 d 19×9
e 45×10 f 18×100 g 36×10 h 41×100

9 How many hours are there in one week? Select the correct answer **A**, **B**, **C** or **D**.

A 84 **B** 151 **C** 168 **D** 240

Worked solutions
Exercise 3-02
MAT07NAWS10013

10 A box contains 124 oranges. If Jessica ordered 21 boxes of oranges, how many oranges will she receive?

Worksheet
Whole numbers 7
MAT07NAWK00011

TLF learning object
Rectangle division (L3704)

Weblink
The Four 4s puzzle

Puzzle sheet
The accidental detective
MAT07NAPS00014

Puzzle sheet
Cross number puzzle
MAT07NAPS10008

3-03 Dividing numbers

Example 6

Evaluate each quotient.

a $318 \div 6$ **b** $1964 \div 5$

Solution

a

$$\begin{array}{r} 5\ 3 \\ 6\overline{)3\ 1^{1}8} \end{array}$$

$318 \div 6 = 53$

b

$$\begin{array}{r} 3\ 9\ \ 2\ \text{r}4 \\ 5\overline{)19^{4}6^{1}4} \end{array}$$

Write the remainder, 4, as a fraction over the divisor, 5:

$1964 \div 5 = 392\frac{4}{5}$

Mental division

Dividing by:	Mental strategy
2, 4 or 8	Halve once, two times or three times respectively
Dividing a multiple of 10 by:	
5	Divide by 10, then double
10	Delete 0 from the end of the number
100	Delete 00 from the end of the number

Example 7

Evaluate each quotient.

a $520 \div 8$ **b** $400 \div 100$ **c** $260 \div 5$ **d** $316 \div 4$

Solution

a $520 \div 8 = 520 \div 2 \div 2 \div 2$

$= 260 \div 2 \div 2$

$= 130 \div 2$

$= 65$

Halve 3 times

$520 \div 2 = 500 \div 2 + 20 \div 2 = 250 + 10$
$= 260$

$260 \div 2 = 200 \div 2 + 60 \div 2 = 100 + 30$
$= 130$

$130 \div 2 = 100 \div 2 + 30 \div 2 = 50 + 15 = 65$

Estimate: $520 \div 8 \approx 520 \div 10 = 52$

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b $4\not{0}\not{0} \div 1\not{0}\not{0} = 4 \div 1 = 4$ — Delete two 0s from the end of the number

c $260 \div 5 = 260 \div 10 \times 2$ — Divide by 10, then double
$= 26 \times 2$
$= 52$ — Estimate: $260 \div 5 \approx 300 \div 5 = 60$

d $316 \div 4 = 316 \div 2 \div 2$ — Halve 2 times
$= 158 \div 2$ — $316 \div 2 = 300 \div 2 + 16 \div 2 = 150 + 8 = 158$
$= 79$ — $158 \div 2 = 150 \div 2 + 8 \div 2 = 75 + 4 = 79$
Estimate: $316 \div 4 \approx 320 \div 4 = 80$

Exercise 3-03 Dividing numbers

1 Evaluate each quotient. See Example 6

a $44 \div 4$ b $68 \div 2$ c $84 \div 7$ d $210 \div 3$
e $105 \div 7$ f $390 \div 5$ g $441 \div 3$ h $861 \div 7$
i $2712 \div 6$ j $1116 \div 9$ k $1980 \div 4$ l $3728 \div 8$

2 Evaluate each quotient, expressing the remainder as a fraction.

a $49 \div 4$ b $688 \div 3$ c $952 \div 6$ d $1815 \div 7$

3 At Westvale Catholic College, there are 135 students in Year 7. If they are placed evenly into five classes, how many students are in each class? Select the correct answer **A**, **B**, **C** or **D**.

A 21 **B** 25 **C** 27 **D** 29

4 Divide a restaurant bill of $204 evenly among six people.

5 Evaluate each quotient mentally. See Example 7

a $320 \div 10$ b $8000 \div 100$ c $60\,000 \div 100$ d $1800 \div 10$
e $410 \div 5$ f $90 \div 5$ g $230 \div 5$ h $600 \div 5$

6 Evaluate each quotient mentally.

a $144 \div 2$ b $232 \div 2$ c $648 \div 2$ d $970 \div 2$
e $216 \div 4$ f $488 \div 4$ g $872 \div 8$ h $208 \div 8$

3-04 Divisibility tests

Worksheet
Divisibility tests
MAT07NAWK10021

Is 2016 divisible by 3? How do you know?
It is often useful to know whether a number is divisible by another number. Numbers that divide evenly (without a remainder) into a bigger number are called the **factors** or **divisors** of the number. **Divisibility tests** are rules for finding whether a number is divisible by specific values.

Skillsheet
Factors and divisibility
MAT07NASS10014

Summary

Divisible by:	Test
2	Last digit 0, 2, 4, 6 or 8
3	Sum of digits divisible by 3
4	Last 2 digits form a number divisible by 4
	Or: Sum of double the tens digit and the units digit divisible by 4
5	Last digit 0 or 5
6	Divisible by 2 and 3
8	Last 3 digits form a number divisible by 8
9	Sum of digits divisible by 9
10	Last digit 0

Video tutorial
Divisibility tests
MAT07NAVT10005

Example 8

Test whether 2016 is divisible by:

a 3 **b** 4 **c** 6 **d** 8

Solution

a Sum of digits $= 2 + 0 + 1 + 6 = 9$, which is divisible by 3.

So 2016 is divisible by 3. (Check: $2016 \div 3 = 672$)

b The last 2 digits of 2016 are 16.

16 is divisible by 4.
So 2016 is divisible by 4.

OR: Sum of double the tens digit and the units digit $= 2 \times 1 + 6 = 8$.

8 is divisible by 4.
So 2016 is divisible by 4. (Check: $2016 \div 4 = 504$)

c 2016 ends in 6, so it is divisible by 2 (even).

2016 is divisible by 3 (from part **a**).
So 2016 is divisible by 6 (because it is divisible by 2 and 3). (Check: $2016 \div 6 = 336$)

d The last 3 digits of 2016 are $016 = 16$.

16 is divisible by 8.
So 2016 is divisible by 8. (Check: $2016 \div 8 = 252$)

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Note:

- If a number is divisible by 6, it must also be divisible by 2 and 3 (because $2 \times 3 = 6$)
- If a number is divisible by 10, it must also be divisible by 2 and 5 (because $2 \times 5 = 10$)
- If a number is divisible by 4, it must also be divisible by 2 (because $2 \times 2 = 4$)
- If a number is divisible by 9, it must also be divisible by 3 (because $3 \times 3 = 9$)
- If a number is divisible by 8, it must also be divisible by 2 and 4 (because $2 \times 4 = 8$)
- If a number is divisible by another whole number, it must also be divisible by the **factors** of that whole number

Exercise 3-04 Divisibility tests

See Example 8

1 Test whether each number is divisible by 3.

a 140 **b** 612 **c** 315 **d** 928
e 209 **f** 525 **g** 132 **h** 1652

2 Test whether each number is divisible by 2.

a 117 **b** 205 **c** 6196 **d** 340

3 Test whether each number is divisible by 6. Note that the numbers in parts **a** to **d** have already been tested for divisibility by 3 in question **1**.

a 140 **b** 612 **c** 315 **d** 928
e 475 **f** 303 **g** 864 **h** 1278

4 Test whether each number is divisible by 4.

a 2040 **b** 518 **c** 365 **d** 242
e 356 **f** 728 **g** 4176 **h** 817

5 Test whether each number is divisible by 9.

a 812 **b** 309 **c** 567 **d** 243
e 837 **f** 462 **g** 6444 **h** 3111

Worked solutions
Exercise 3-04
MAT07NAWS10014

6 Test whether 1964 is divisible by 8.

7 Explain by just looking at the last digit why:

a 409 is not divisible by 4 **b** 316 is not divisible by 5
c 2015 is not divisible by 8 **d** 343 is not divisible by 6
e 472 is not divisible by 10 **f** 511 is not divisible by 12

8 Which number is divisible by both 4 and 5? Select the correct answer **A**, **B**, **C** or **D**.

A 10 **B** 15 **C** 20 **D** 25

9 If a number is divisible by 3 and 4, what other number must it also be divisible by?

10 Write a number between 50 and 100 which is divisible by:

a 3 and 5 **b** 4 and 5 **c** 6 and 7 **d** 2 and 6

Extension: Harder divisibility tests

11 There are several complicated divisibility tests for 7. Here is the most popular one:

- Remove the last digit of the number and double it
- Subtract it from the rest of the number
- If the difference is 0 or divisible by 7, the number is divisible by 7

For example, to test 112:

Last digit = 2, double 2 = 4
11 − 4 = 7, which is divisible by 7
So 112 is divisible by 7.

Test whether each number is divisible by 7.

a 357 **b** 1013 **c** 6258 **d** 901

12 Here is a divisibility test for 11.

- Add the digits in the odd places of the number
- Add the digits in the even places of the number
- Subtract the two sums
- If the difference is 0 or divisible by 11, the number is divisible by 11

For example, to test 2016:

Add digits in odd places: 2 + 1 = 3
Add digits in even places: 0 + 6 = 6
6 – 3 = 3, which is not divisible by 11
So 2016 is not divisible by 11.

Test whether each number is divisible by 11.

a 814 **b** 2728 **c** 4051 **d** 6105

Mental skills 3A Maths without calculators

Multiplying by a multiple of 10

Place value allows us to simply add zeros to the end of a number whenever we multiply by a power of 10.

1 Consider these examples.

a 37 × 10 = 370
b 45 × 100 = 4500
c 16 × 1000 = 16 000
d 100 × 1000 = 100 000
e 7 × 90 = 7 × 9 × 10 = 63 × 10 = 630
f 5 × 400 = 5 × 4 × 100 = 20 × 100 = 2000
g 12 × 300 = 12 × 3 × 100 = 36 × 100 = 3600
h 40 × 800 = 4 × 10 × 8 × 100 = 4 × 8 × 10 × 100 = 32 × 1000 = 32 000

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2 Now evaluate each product.

a 18 × 100	**b** 26 × 1000	**c** 77 × 10 000	**d** 10 × 100
e 315 × 1000	**f** 1000 × 1000	**g** 294 × 10	**h** 475 × 100
i 3 × 80	**j** 8 × 200	**k** 6 × 50	**l** 7 × 30
m 2 × 6000	**n** 11 × 900	**o** 4 × 400	**p** 5 × 700
q 5 × 80	**r** 25 × 20	**s** 300 × 60	**t** 900 × 4000

3-05 Long division

NSW

Worksheet
Four operations
MAT07NAWK00020

Long division is a technique for dividing by number with two or more digits, that is, a number greater than 10.

Example 9

Evaluate each quotient.

a 312 ÷ 12 **b** 296 ÷ 21

Solution

a

$$\begin{array}{r} 26 \\ 12\overline{)312} \\ -24\downarrow \\ \hline 72 \\ -72 \\ \hline 0 \end{array}$$

12 into 31 is 2, remainder 7

12 into 72 is 6

Or

$$\begin{array}{rl} 26 & \\ 12\overline{)312} & \\ -120 & \text{10 times} \\ \hline 192 & \\ -120 & \text{10 times} \\ \hline 72 & \\ -72 & \text{6 times} \\ \hline 0 & \text{26 times} \end{array}$$

Guessing with 'easy' multiples of 12

312 ÷ 12 = 26.

Check by estimating: 312 ÷ 12 ≈ 300 ÷ 10 = 30

b

$$\begin{array}{r} 14 \\ 21\overline{)296} \\ -21\downarrow \\ \hline 86 \\ -84 \\ \hline 2 \end{array}$$

21 into 29 is 1, remainder 8

21 into 86 is 4, remainder 2

Or

$$\begin{array}{rl} 14 & \\ 21\overline{)296} & \\ -210 & \text{10 times} \\ \hline 86 & \\ -84 & \text{4 times} \\ \hline 2 & \text{14 times} \end{array}$$

$296 \div 21 = 14\frac{2}{21}$

Write the remainder as the numerator of a fraction

Check by estimating: $296 \div 21 \approx 300 \div 20 = 15$

Exercise 3-05 Long division

See Example 9

1 Use long division to evaluate each quotient.

a $180 \div 15$ **b** $462 \div 22$ **c** $731 \div 17$
d $275 \div 11$ **e** $2352 \div 16$ **f** $78 \div 13$
g $900 \div 25$ **h** $667 \div 23$ **i** $6848 \div 32$

2 Evaluate each quotient, writing the remainder as a fraction.

a $304 \div 12$ **b** $505 \div 14$ **c** $990 \div 26$

3 At a party 275 lollies are shared equally among 25 children. How many lollies does each child get?

4 A piece of wood 390 cm in length is to be cut into 15 equal pieces. How long is each piece?

5 Mrs Kaur needs $1550 to purchase a new LED television. If she can save $62 each week, how long will it take her to save enough money to purchase the television?

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3-06 Roman numerals

NSW

Puzzle sheet
Roman numerals dominoes
MAT07NAPS10010

Skillsheet
Roman numerals
MAT07NASS10015

Weblink
Number systems

In 500 BCE, most of Europe was ruled by the Roman Empire, so Roman numerals were widely used until the end of the 16th century. Roman numerals use letters of the alphabet and are still often used today.

The ancient Romans used the following numerals:

1	2	3	4	5	6	7	8	9	10
I	II	III	IV	V	VI	VII	VIII	IX	X

20	30	40	50	60	90	100	400	500	1000
XX	XXX	XL	L	LX	XC	C	CD	D	M

The Romans had an unusual method of writing numbers involving 4 or 9, to avoid writing the same letter more than 3 times in a row. A smaller number written before a larger number meant a subtraction ('minus').

- Instead of writing 4 as IIII, they wrote **IV** meaning V – I (that is 5 – 1 = 4).
- Instead of writing 9 as VIIII they wrote **IX** meaning X – I (that is 10 – 1 = 9).
- For 40, they wrote **XL** (that is 50 – 10 = 40).
- For 90, they wrote **XC** (that is 100 – 10 = 90).

Example 10

Write each number in Roman numerals.

a 23 **b** 46 **c** 279

Solution

a 23 = 20 + 3
= XX + III
= XXIII

b 46 = 40 + 6
= XL + VI
= XLVI

c 279 = 200 + 70 + 9
= CC + LXX + IX
= CCLXXIX

Example 11

Write each Roman numeral as a number.

a LXXIII **b** CI **c** CDXXVIII

Solution

a LXXIII = LXX + III
= 70 + 3
= 73

b CI = C + I
= 100 + 1
= 101

c CDXXVIII = CD + XX + VIII
= 400 + 20 + 8
= 428

Exercise 3-06 Roman numerals

See Example 10

1 Write each number using Roman numerals.

a 27 **b** 52 **c** 79 **d** 93
e 2600 **f** 314 **g** 928 **h** 3476
i 165 **j** 230 **k** 2482 **l** 109

See Example 11

2 Titus, a student in ancient Rome, wrote these numerals. Rewrite them as numbers.

a XXVI **b** XL **c** CCLXIV **d** LIV
e LXXVII **f** XLV **g** CCCXXXIX **h** DXXVIII
i MMCLXII **j** MCMXC **k** XCVIII **l** MDVII

3 **a** Where do you see Roman numerals being used today?
b Why do you think Roman numerals are no longer widely used?

4 The Roman word for hundred was 'centum' which is why C stands for 100. List some words beginning with 'cent' that mean one hundred of something.

5 **a** Write this year in Roman numerals.
b Which year of the 21st century will use the most letters when written in Roman numerals?

6 Find out what the Roman numerals $\overline{\text{V}}$ and $\overline{\text{X}}$ mean.

Investigation: Aboriginal numbers

Traditionally, indigenous Australians had no need for a complicated number system in their everyday life. Aboriginal society relied on story-telling, using the spoken language rather than writing, and Aboriginal people did not have symbols for numbers. Different regions had their own names for numbers.

The **Belyando River people** of central Queensland used only two words to name their numbers:

1 = wogin 2 = booleroo
3 = booleroo wogin 4 = booleroo booleroo

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The **Kamilaroi people** lived in northern New South Wales, including the regions surrounding Moree and Tamworth. They used three words to name their numbers.

1 = mal	2 = bularr	3 = guliba
4 = bularr bularr	5 = bularr guliba	6 = guliba guliba

1 How did the Belyando River people form words for the numbers 3 and 4?
2 How did the Kamilaroi people form words for 4, 5 and 6?
3 Write the answer to each expression, using the correct Aboriginal words:

a wogin + booleroo wogin
b guliba × bularr
c bularr + bularr + mal
d booleroo × booleroo
e guliba guliba – guliba
f bularr bularr – mal

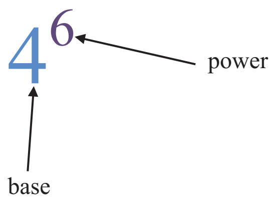
4 State one advantage and one disadvantage of working with Aboriginal numbers.

3-07 Powers and index notation

The use of powers allows us to write repeated multiplication in a shorter way.

$4^2 = 4 \times 4$	'4 squared' or '4 to the power of 2'
$4^3 = 4 \times 4 \times 4$	'4 cubed' or '4 to the power of 3'
$4^4 = 4 \times 4 \times 4 \times 4$	'4 to the power of 4'
$4^5 = 4 \times 4 \times 4 \times 4 \times 4$	'4 to the power of 5'
$4^6 = 4 \times 4 \times 4 \times 4 \times 4 \times 4$	'4 to the power of 6'

In 4^6, the number 4 is called the **base** and is the number that is repeated in the multiplication. The small raised number 6 is called the **power** or **index**.



Example 12

Write each expression using index notation.

a $6 \times 6 \times 6 \times 6 \times 6$
b $7 \times 7 \times 7 \times (-2) \times (-2) \times (-2) \times (-2)$

Solution

a $6 \times 6 \times 6 \times 6 \times 6 = 6^5$
b $7 \times 7 \times 7 \times (-2) \times (-2) \times (-2) \times (-2) = 7^3 \times (-2)^4$

Example 13

Evaluate each expression.

a 11^2 **b** $(-3)^3$ **c** 2^6

Solution

a $11^2 = 11 \times 11 = 121$

b $(-3)^3 = (-3) \times (-3) \times (-3) = -27$

c $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

Exercise 3-07 Powers and index notation

See Example 12

1 Write each expression using index notation.

a $8 \times 8 \times 8 \times 8$
b $7 \times 7 \times 7 \times 7 \times 7 \times 7$
c $-5 \times (-5) \times (-5)$
d $1 \times 1 \times 1 \times 1 \times 1$
e $13 \times 13 \times 13 \times 13$
f 10
g $(-4) \times (-4) \times 9 \times 9 \times 9$
h $2 \times 2 \times 12 \times 12$
i $3 \times 3 \times 11 \times 11 \times 11 \times 11$
j $15 \times (-6) \times (-6) \times 15$
k $2 \times 8 \times 2 \times 8 \times 2$
l $10 \times 10 \times 20$

2 Write each expression in expanded form (for example, $2^4 = 2 \times 2 \times 2 \times 2$).

a 3^5
b 5^2
c 7^4
d $(-6)^3$
e $2^4 \times 4^2$
f $12^3 \times 11^6$

See Example 13

3 Evaluate each expression.

a 4^2
b 5^3
c 2^5
d 10^6
e 3^5
f 7^4
g 1^3
h $(-2)^4$
i 3^3
j $(-1)^2$
k $(-9)^5$
l 4^6

Worksheet
Big numbers
MAT07NAWK00019

4 Copy and complete this table of powers of 10.

Power of 10	Number	Name
10^1	10	ten
10^2	100	hundred
10^3	1000	
10^6	1 000 000	
10^9		billion
10^{12}	1 000 000 000 000	
10^{15}		quadrillion
	1 000 000 000 000 000 000	quintillion
10^{21}		sextillion
10^{24}		septillion
	1 000 000 000 000 000 000 000 000 000	octillion
10^{30}		nonillion
10^{33}	1 000 000 000 000 000 000 000 000 000 000 000	decillion

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Technology Powers

1 Set up a spreadsheet to calculate powers by first entering the information shown below.

	A	B	C	D	E
1	Number:	2	3	5	7
2	1				
3	2				
4	3				
5	4				
6	5				
7	6				
8	7				
9	8				
10	9				
11	10				
12	11				
13	12				

2 In cell B2, enter = **B1^A2** to calculate 2^1, where **^** in a spreadsheet formula means 'to the power of'. We also use **B1** instead of **B1** because when we copy this formula, we want it to **always** refer to the **2** in cell B1, not another cell. This is called **absolute cell referencing**. We use this technique to maintain a particular value in a cell without changing it when writing a formula. However, we use **A2** instead of **A2** because when we copy this formula, we want it to refer to the different powers shown in column A to calculate 2^1, 2^2, 2^3, and so on. This is called **relative cell referencing**.

3 Click on cell B2 and Fill Down to cell B13. Your spreadsheet will now show the first 12 powers of 2, up to $2^{12} = 4096$.

4 Enter similar formulas for cells C2, D2 and E2, and **Fill Down** for columns C, D and E to show the first 12 powers of 3, 5 and 7.

5 If a number is too long to show in a cell, it will either look something like **2.44E+08** or **########**. For cells that show something like **2.44E+08**, right-click, select **Format cells**, **Number** and **0** for decimal places. **########** means the column is not wide enough to show all digits of the number. You need to widen the column until you can see all digits.

Just for the record Googol vs Google

The number 10^{100}, the **googol,** is 1 followed by one hundred zeros. The name 'googol' was created by the 9-year-old nephew of American mathematician Dr Edward Kasner. The number 10^{googol}, that is 1 followed by a googol zeros, is called the **googolplex**. The googol is a very big number but it is rarely used for practical purposes. Even the number of particles in the observable universe, estimated at being between 10^{72} and 10^{87}, is less than a googol!

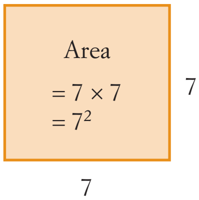
The Internet search engine Google was named after the googol, to reflect the huge size of the World Wide Web. It was created in 1996 by two Stanford University students, Larry Page and Sergey Brin. They even named Google's global headquarters in California the Googleplex. Google is a powerful search engine because it can find information from over a trillion (a million million) web pages in less than 1 second.

How many googols are there in a googolplex?

3-08 Square root and cube root

Square and square root

Squaring a number means raising it to the **power of 2**.
For example, 'squaring 7' or 'the square of 7' means $7^2 = 7 \times 7 = 49$, read '7 squared'. Also, 49 is called a **square number** or **perfect square** because it is the square of a whole number.
It is called '7 squared' because it is the area of a square of side 7 units.
The opposite of squaring is the **square root** (symbol $\sqrt{\ }$).
For example, $\sqrt{49} = 7$, read 'the square root of 49', because $7^2 = 7 \times 7 = 49$.



Worksheet
Powers and roots
MAT07NAWK10022

Summary

The **square root** $\sqrt{\ }$ of a number is the positive value which, when squared, gives that number.

Puzzle sheet
Square root Snap
MAT07NAPS10011

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Skillsheet
Square roots and cube roots
MAT07NASS10016

Homework sheet
Powers and square root
MAT07NAHS10002

Example 14

Evaluate:

a the square root of 64 **b** $\sqrt{9}$ **c** $\sqrt{25}$

Solution

a The square root of 64 $= \sqrt{64} = 8$ because $8^2 = 8 \times 8 = 64$

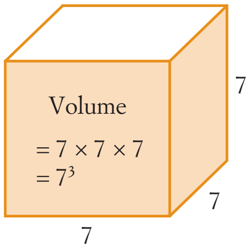
b $\sqrt{9} = 3$ because $3^2 = 3 \times 3 = 9$

c $\sqrt{25} = 5$ because $5^2 = 5 \times 5 = 25$

Cube and cube root

Cubing a number means raising it to the **power of 3**.

For example, 'cubing 7' or 'the cube of 7' means $7^3 = 7 \times 7 \times 7 = 343$, read '7 cubed'. Also, 343 is called a **cube number** or **perfect cube** because it is the cube of a whole number.



It is called '7 cubed' because it is the volume of a cube of side 7 units.

The opposite of cubing is the **cube root** (symbol $\sqrt[3]{\ }$).

For example, $\sqrt[3]{343} = 7$, read 'the cube root of 343', because $7^3 = 7 \times 7 \times 7 = 343$.

Summary

The **cube root** $\sqrt[3]{\ }$ of a number is the value which, when cubed, gives that number.

Example 15

Evaluate:

a the cube root of 125 **b** $\sqrt[3]{8}$ **c** $\sqrt[3]{729}$

Solution

a The cube root of 125 $= \sqrt[3]{125} = 5$ because $5^3 = 5 \times 5 \times 5 = 125$

b $\sqrt[3]{8} = 2$ because $2^3 = 2 \times 2 \times 2 = 8$

c $\sqrt[3]{729} = 9$ because $9^3 = 9 \times 9 \times 9 = 729$

Example 16

Estimate the value of $\sqrt{40}$.

Solution

There is no *exact* answer for the square root of 40, because there isn't a number which, if squared, equals 40 exactly. However, we can find a decimal whose square is close to 40.

Noting that:

$5^2 = 25$

$6^2 = 36$

$7^2 = 49$

we can tell that $\sqrt{40}$ must lie somewhere between 6 and 7, and closer to 6 because 40 is closer to 36 than 49.

So we can estimate that $\sqrt{40} \approx 6.3$.

(In fact, $6.3^2 = 39.69$).

Exercise 3-08 Square root and cube root

Extra questions
Index notation
MAT07NAEQ00011

1 Evaluate each square number.

a 9^2 **b** 17^2 **c** 10^2

2 The x^2 key on a calculator can be used to square a number. For example, to find 9^2, press 9 x^2 =. Use your calculator to evaluate each square number.

a 7^2 **b** 13^2 **c** 18^2
d 1^2 **e** 5^2 **f** 20^2
g 16^2 **h** 34^2 **i** 15^2

See Example 14

3 Use your answers from question **2** to find:

a $\sqrt{400}$ **b** $\sqrt{225}$ **c** $\sqrt{324}$
d $\sqrt{256}$ **e** $\sqrt{169}$ **f** $\sqrt{1156}$

4 Copy and complete this table.

Number, x	1	2	3	4	5	6	7	8	9	10	11	12
Number squared, x^2												

5 Which one of the following numbers is a square number? Select **A**, **B**, **C** or **D**.

A 32 **B** 36 **C** 40 **D** 45

6 Use your answers from the table in question **4** to find:

a the square root of 100 **b** $\sqrt{36}$ **c** $\sqrt{121}$
d $\sqrt{16}$ **e** $\sqrt{1}$ **f** the square root of 81
g $\sqrt{64}$ **h** the square root of 144 **i** $\sqrt{25}$

7 There are two numbers whose square root is itself, that is, $\sqrt{x} = x$. What are the two numbers?

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8 The [√] key on a calculator can be used to find the square root of a number. For example, to find $\sqrt{64}$, press [√] 64 [=]. Use your calculator to find each square root.

a $\sqrt{49}$ **b** $\sqrt{81}$ **c** $\sqrt{625}$
d $\sqrt{484}$ **e** $\sqrt{1764}$ **f** $\sqrt{361}$
g $\sqrt{900}$ **h** $\sqrt{784}$ **i** $\sqrt{256}$
j $\sqrt{196}$ **k** $\sqrt{400}$ **l** $\sqrt{3136}$

9 Evaluate each cube number.

a 4^3 **b** 11^3 **c** 1^3

10 The [x^3] key on a calculator can be used to cube a number. For example, to find 4^3, press 4 [x^3] [=]. Use your calculator to find each cube number.

a 7^3 **b** 16^3 **c** 5^3
d 10^3 **e** 20^3 **f** 17^3
g 35^3 **h** $(-8)^3$ **i** $(-5)^3$

11 Copy and complete this table.

Number, x	1	2	3	4	5	6	7	8	9	10	11	12
Number cubed, x^3												

12 Which of the following numbers is a cube number? Select **A**, **B**, **C** or **D**.

A 192 **B** 512 **C** 625 **D** 800

13 Use your answers from question **11** to find: See Example 15

a the cube root of 512 **b** $\sqrt[3]{729}$ **c** $\sqrt[3]{125}$
d $\sqrt[3]{1}$ **e** $\sqrt[3]{27}$ **f** the cube root of 1728
g $\sqrt[3]{64}$ **h** the cube root of 1331 **i** $\sqrt[3]{1000}$

14 Use your answers from question **10** to find:

a $\sqrt[3]{-512}$ **b** $\sqrt[3]{8000}$ **c** $\sqrt[3]{42\,875}$
d $\sqrt[3]{-125}$ **e** $\sqrt[3]{4913}$ **f** $\sqrt[3]{4096}$

15 The [$\sqrt[3]{\ }$] key on a calculator can be used to find the cube root of a number. For example, to find $\sqrt[3]{64}$, press [$\sqrt[3]{\ }$] 64 [=]. Use your calculator to find each cube root.

a $\sqrt[3]{4096}$ **b** $\sqrt[3]{2744}$ **c** $\sqrt[3]{10\,648}$
d $\sqrt[3]{64\,000}$ **e** $\sqrt[3]{19\,683}$ **f** $\sqrt[3]{9261}$

16 Use your answers from the table in question **4** to determine between which two consecutive whole numbers $\sqrt{27}$ must lie. Select the correct answer **A**, **B**, **C** or **D**. See Example 16

A 26 and 28 **B** 13 and 14 **C** 4 and 5 **D** 5 and 6

17 Determine between which two consecutive whole numbers $\sqrt{15}$ must lie. Select the correct answer **A**, **B**, **C** or **D**.

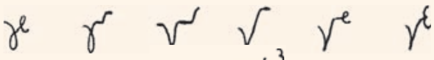
A 7 and 8 **B** 3 and 4 **C** 196 and 197 **D** 4 and 5

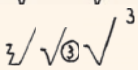
18 Estimate the value of each root, then use your calculator to check.

a $\sqrt{50}$ **b** $\sqrt{32}$ **c** $\sqrt{96}$
d $\sqrt[3]{71}$ **e** $\sqrt[3]{900}$ **f** $\sqrt[3]{184}$

Just for the record — The radical symbol

The formal name for the square root symbol ($\sqrt{\ }$) is the **radical symbol.** This symbol was created by the German mathematician Christoff Rudolff in 1525, but it was not until the end of the 17th century that the symbol was widely accepted. Before this, the most commonly-used symbol for square root was ℞. It was first used in 1220 and was an abbreviation for the word **radix**, which means "root" in Latin. One theory says that Rudolff invented the symbol $\sqrt{\ }$ to look like the letter 'r'.

Other early symbols for square root were 

The symbols on the right all represent the same thing. 

What do you think they stand for?

Mental skills 3B — Maths without calculators

Dividing by a multiple of 10

Place value allows us to remove zeros from the end of a number when we divide by a power of 10.

1 Study each example.

a 2000 ÷ 10 = 200~~0~~ ÷ 1~~0~~ = 200
b 1800 ÷ 100 = 18~~00~~ ÷ 1~~00~~ = 18
c 37 000 ÷ 100 = 37 0~~00~~ ÷ 1~~00~~ = 370
d 5 000 000 ÷ 1000 = 5 000 ~~000~~ ÷ 1~~000~~ = 5000
e 6000 ÷ 200 = 60~~00~~ ÷ 1~~00~~ ÷ 2 = 60 ÷ 2 = 30
f 350 ÷ 70 = 35~~0~~ ÷ 1~~0~~ ÷ 7 = 35 ÷ 7 = 5
g 2800 ÷ 40 = 280~~0~~ ÷ 1~~0~~ ÷ 4 = 280 ÷ 4 = 70
h 40 000 ÷ 5000 = 40 ~~000~~ ÷ 1~~000~~ ÷ 5 = 40 ÷ 5 = 8

2 Now evaluate each quotient.

a 200 ÷ 10 **b** 6000 ÷ 100 **c** 45 000 ÷ 100 **d** 30 000 ÷ 1000
e 1900 ÷ 10 **f** 2600 ÷ 100 **g** 530 ÷ 10 **h** 720 000 ÷ 1000
i 180 ÷ 30 **j** 300 ÷ 50 **k** 1600 ÷ 400 **l** 45 000 ÷ 5000
m 4200 ÷ 60 **n** 21 000 ÷ 700 **o** 44 000 ÷ 2000 **p** 1600 ÷ 200
q 24 000 ÷ 600 **r** 15 000 ÷ 3000 **s** 64 000 ÷ 80 **t** 5400 ÷ 900

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3-09 Prime and composite numbers

The **factors** of a number are those whole numbers that divide exactly into it (without remainder). For example:

- The factors of 16 are 1, 2, 4, 8 and 16
- The factors of 7 are 1 and 7

A number is **prime** if it has only **two** factors: 1 and itself. Some examples of prime numbers are 7, 3, 19, 2 and 11.

A number is **composite** if it has **more than two** factors. Some examples of composite numbers are 16, 10, 9, 15 and 4.

Worksheet
Sieve of Eratosthenes
MAT07NAWK10023

Summary

- A **prime number** has only two factors: 1 and itself
- A **composite number** has more than two factors

Note: The number 1 has only one factor so it is neither prime nor composite.

Skillsheet
Prime and composite numbers
MAT07NASS10017

Exercise 3-09 Prime and composite numbers

1 List all the factors of each number

a 17 **b** 21 **c** 24 **d** 11
e 35 **f** 4 **g** 18 **h** 23
i 25 **j** 9 **k** 3 **l** 19

2 List all the numbers from question **1** that are:

a prime **b** composite

3 Which one of these numbers is not a factor of 45? Select the correct answer **A**, **B**, **C** or **D**.

A 9 **B** 5 **C** 7 **D** 3

4 Sort the following numbers into primes and composites.

10 2064 101 129 33 77 967 504
999 472 17 27 41 71 62 59

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5 The ancient Greek mathematician, Eratosthenes, discovered an easy way to sort out the prime numbers from a list of numbers. It is called the Sieve of Eratosthenes (pronounced 'Siv of Era-tos-the-nees'), and works by crossing out multiples of numbers (the composite numbers).

a Copy the grid below for 1 to 120 or print out the Worksheet 'Sieve of Eratosthenes'.

Worksheet
Sieve of Eratosthenes
MAT07NAWK10023

TLF learning object
Sieve of Eratosthenes (L3545)

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60
61	62	63	64	65	66
67	68	69	70	71	72
73	74	75	76	77	78
79	80	81	82	83	84
85	86	87	88	89	90
91	92	93	94	95	96
97	98	99	100	101	102
103	104	105	106	107	108
109	110	111	112	113	114
115	116	117	118	119	120

b Cross out 1. It is neither prime nor composite.

c Except for 2, colour all the multiples of 2 red and notice the pattern.

d Except for 3, colour all the multiples of 3 green and notice the pattern.

e Continue to colour multiples of other numbers with different colours, until there are no more multiples.

f What do you notice about the 30 numbers left that are not coloured?

6 List all the:

a prime numbers between 36 and 50

b composite numbers between 65 and 80

c prime numbers less than 20

d composite numbers between 30 and 47

Worked solutions
Exercise 3-09
MAT07NAWS10015

7 Which one of the following numbers is divisible only by prime numbers, itself and 1? Select **A**, **B**, **C** or **D**.

A 12 **B** 14 **C** 16 **D** 18

8 Look up other meanings for the word 'composite'. Suggest why this word is used the way it is in mathematics.

9 **a** Find a number that has an odd number of factors.

b What type of numbers have an odd number of factors?

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3-10 Prime factors

Every number can be written as a product of its prime factors. The prime factors can be found by using a **factor tree**.

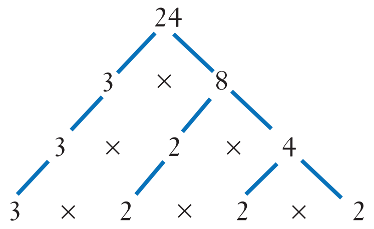
Worksheet
Factor trees
MAT07NAWK10024

Example 17

Write 24 as a product of its prime factors.

Solution

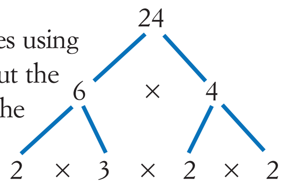
Draw a factor tree for 24.



- 8 is composite, so write 8 as a product of 2 factors.
- 3 is prime, so keep writing 3 along the branches.

So 24 as a product of prime factors is:
$24 = 2 \times 2 \times 2 \times 3$.

Note: It is possible to draw different factor trees using different factors for the same number, but the final list of prime factors should still be the same. Here is another factor tree for 24:



$24 = 2 \times 2 \times 2 \times 3$.

Video tutorial
Factor trees
MAT07NAVT10006

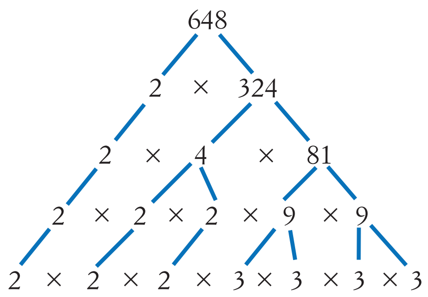
Skillsheet
Prime factors by repeated division
MAT07NASS10018

Example 18

Write 648 as a product of its prime factors, using index notation.

Solution

Draw a factor tree for 648.



- 2 is prime, so keep writing 2 along the branches.
- 324 is composite, so write 324 as a product of 2 factors, 4 and 81.
- Keep going until there are no composite factors left.

So 648 as a product of prime factors is:

$$648 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$$
$$= 2^3 \times 3^4$$

Finding square and cube roots using prime factors

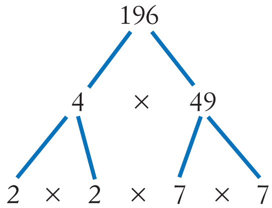
Example 19

Use a factor tree to find the value of:

a $\sqrt{196}$ **b** $\sqrt[3]{216}$

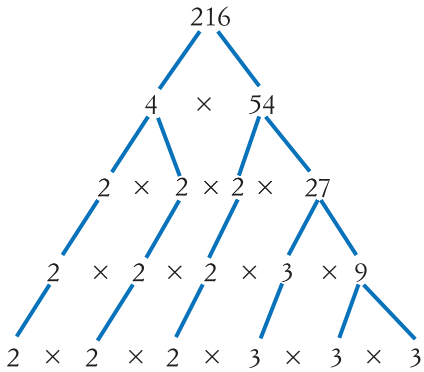
Solution

a Draw a factor tree for 196.



$$\text{So } 196 = 2 \times 2 \times 7 \times 7$$
$$\therefore \sqrt{196} = \sqrt{2 \times 2 \times 7 \times 7}$$
$$= 2 \times 7 \qquad \sqrt{2 \times 2} = 2,\ \sqrt{7 \times 7} = 7$$
$$= 14$$

b Draw a factor tree for 216.



$$\text{So } 216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$$
$$\therefore \sqrt[3]{216} = \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3}$$
$$= 2 \times 3 \qquad \sqrt[3]{2 \times 2 \times 2} = 2,\ \sqrt[3]{3 \times 3 \times 3} = 3$$
$$= 6$$

Exercise 3-10 Prime factors

See Example 17

1 Use a factor tree to write each number as a product of its prime factors.

a 88	**b** 63	**c** 45	**d** 51
e 132	**f** 270	**g** 396	**h** 218
i 630	**j** 520	**k** 275	**l** 342

2 What is 1260 expressed as a product of its prime factors? Select **A**, **B**, **C** or **D**.

A $2 \times 3 \times 3 \times 3 \times 5 \times 7$ **B** $2 \times 2 \times 2 \times 3 \times 5 \times 7$
C $2 \times 2 \times 3 \times 3 \times 3 \times 7$ **D** $2 \times 2 \times 3 \times 3 \times 5 \times 7$

3 Draw two different factor trees for 280 and show that both give the same prime factors.

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4 Use a factor tree to write each number as a product of its prime factors, in index notation. See Example 18

a 48 **b** 200 **c** 460 **d** 712
e 98 **f** 144 **g** 325 **h** 135

5 484 is a perfect square. See Example 19

a Use a factor tree to write 484 as a product of its prime factors.

b Note that because 484 is a square number, the prime factors are in pairs. Use the factor tree to find $\sqrt{484}$.

6 Use a factor tree to evaluate each root.

a $\sqrt{625}$ **b** $\sqrt{900}$ **c** $\sqrt{225}$ **d** $\sqrt{784}$
e $\sqrt[3]{9261}$ **f** $\sqrt[3]{3375}$ **g** $\sqrt[3]{1728}$ **h** $\sqrt[3]{4096}$

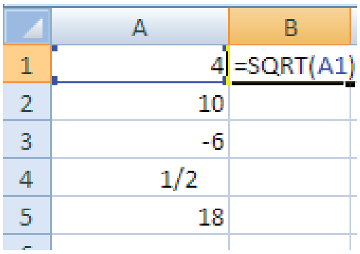
Worked solutions
Exercise 3-10
MAT07NAWS10016

Technology Spreadsheet formulas

Remember that a spreadsheet is like a calculator. We use special symbols to make calculations and a spreadsheet formula always begins with an equals (=) sign.

Formula	Meaning
=A1+A2+A3 or **=sum(A1:A3)**	**add** the values in cells A1, A2 and A3
=A5–A4	**subtract** the value in cell A4 from the value in cell A5
=A1*A3	**multiply** together the values in cells A1 and A3 (* is used instead of ×)
=A1/A2	**divide** the value in cell A1 by the value in cell A2 (/ is used instead of ÷)
=A2^2	**square** the value in cell A2 (^ is used instead of $(A2)^2$)
=A5^3	**cube** the value in cell A5, that is, $(A5)^3$
=sqrt(A3)	find the **square root** of the value in cell A3, that is, $\sqrt{A3}$
=average(A1:A5)	find the **average** of all values from cells A1 to A5
=max(A1:A8)	find the **maximum** (**highest**) of all values from cells A1 to A8
=min(A1:A8)	find the **minimum** (**lowest**) of all values from cells A1 to A8

1 Open a new spreadsheet and enter the 5 numbers shown in column A. To enter the fraction **1/2** in cell B4, right-click on B4 to select **Format Cells**, then choose **Fraction** and select **Up to three digits**.



	A	B
1	4	=SQRT(A1)
2	10	
3	-6	
4	1/2	
5	18	

2 In cell B1, enter the formula **=sqrt(A1)** to calculate the square root of the value in cell A1. You should get the answer 2, because $\sqrt{4} = 2$.

3 For the rest of the cells in column B, enter the appropriate formula for each expression shown below and check that the spreadsheet calculates the correct answer. Remember to start each formula with an '=' sign.

a at B2, A2 ÷ A1 **b** at B3, $(A3)^2$
c at B4, $(A3)^3$ **d** at B5, A1 × A2 + A3 × A4

e at B6, A5 – A3
f at B7, the sum of A1 to A5
g at B8, the product of A1 to A5
h at B9, $(A4)^2$
i at B10, $(A5)^3$
j at B11, $\sqrt{\frac{A5}{A4}}$

4 For the following cells in column C, enter the appropriate formula for each expression and check that the spreadsheet calculates the correct answer.
   a at C1, the average of column A
   b at C2, the maximum value of column A
   c at C3, the minimum value of column A
   d at C4, $(A1 + A5)^2$
   e at C5, the average of A1, A2 and A3
   f at C6, $\sqrt{A2 - A3}$

5 Now enter **new** values in cells A1 to A5 (including a new fraction for A4) and notice the new answers calculated in columns B and C.

6 Create your own values and formulas. Write at least five formulas and use up to five different values. Enter the new values in column D and the new formulas in column E of your spreadsheet.

3-11 Greatest common divisor

Summary

The **greatest common divisor** (GCD) or **highest common factor** (HCF) of two (or more) numbers is the largest number that is a factor of **both** (**or all**) of these numbers.

Divisor is just another name for **factor**.

Example 20

Animated example
Highest common factors
MAT07NAAE00012

Find the greatest common divisor (GCD) of:

a 24 and 30
b 40 and 15

Solution

a Factors of 24 = **1**, **2**, **3**, 4, **6**, 8, 12, 24
   Factors of 30 = **1**, **2**, **3**, 5, **6**, 10, 15, 30
   The common factors are **1**, **2**, **3**, **6**
   The greatest common divisor is 6.

b Factors of 40 = **1**, 2, 4, **5**, 8, 10, 20, 40
   Factors of 15 = **1**, 3, **5**, 15
   The greatest common divisor is 5.

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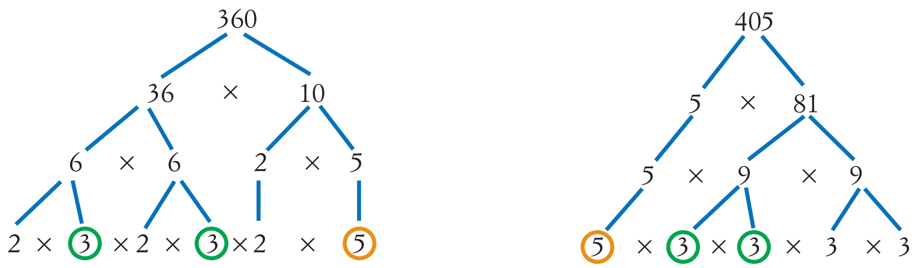
Using prime factors to find the GCD

Example 21

Use factor trees to find the GCD of 360 and 405.

Solution



- Draw factor trees for 360 and 405
- Circle common prime factors: 3, 3 and 5
- Multiply 3, 3 and 5 to calculate the GCD

$$\begin{aligned} \text{GCD} &= 3 \times 3 \times 5 \\ &= 45 \end{aligned}$$

45 is the highest factor of both 360 and 405

Summary

To find the greatest common divisor of two numbers using their prime factors:

1 Circle common prime factors
2 Multiply them together

Exercise 3-11 Greatest common divisor

1 Which one of these numbers is **not** a factor of 36? Select **A**, **B**, **C** or **D**.

A 4 **B** 6 **C** 8 **D** 9

2 Find all common factors for each pair of numbers.

a 27, 45 **b** 60, 32 **c** 75, 45 **d** 18, 28

3 Find the greatest common divisor of each pair of numbers. See Example 17

a 32, 28 **b** 9, 6 **c** 6, 14 **d** 8, 12
e 14, 70 **f** 44, 64 **g** 10, 15 **h** 12, 6
i 75, 125 **j** 56, 40 **k** 60, 90 **l** 39, 26
m 132, 60 **n** 36, 84 **o** 27, 63 **p** 350, 210

4 On a spreadsheet, the greatest common divisor can be found using the function =**gcd()**.

a Enter =**gcd(27, 45)** into a spreadsheet cell to find the GCD of 27 and 45.

b Check your answers to question **3** using a spreadsheet.

See Example 18

5 Use factor trees to find the greatest common divisor of each pair of numbers.

a 324, 486 **b** 144, 60 **c** 35, 210 **d** 1404, 1900

Investigation: Common factor puzzle

1 Copy this grid, and try to move from the 200 square in the top left corner to the 100 square at the bottom right corner. You can move one step horizontally or vertically (but not diagonally), but only if both numbers have a common factor (not 1). For example, you can move from 65 to 91 (common factor 13), but not to 96 (no common factor).

200	80	65	91	143	156	195
175	32	96	71	110	77	121
35	28	15	209	87	90	21
39	169	117	95	57	37	81
63	11	29	72	76	75	51
14	98	56	132	48	78	85
105	45	44	187	112	221	100

2 Now try to move from the 105 square in the bottom left corner to the 195 square at the top right corner, using the same rules.

3 Choose different starting and finishing positions. Do they all have connecting factor paths?

Investigation: GCD by repeated division

The greatest common divisor can be found by repeated division with prime factors 2, 3, 5, 7, and so on. For example, to find the GCD of 24 and 60:

- both numbers are even so divide by 2 → 2)**24** 2)**60**
- both numbers are even so divide by 2 again → 2)**12** 2)**30**
- both numbers are divisible by 3 so divide by 3 → 3)**6** 3)**15**
- Since it is not possible to divide any more, stop. 2 5
- Multiply the common prime factors to find the GCD.
- GCD of 24 and 60 = 2 × 2 × 3 = 12

Use this method to find the GCD of each set of numbers.

a 45, 120 **b** 18, 48 **c** 20, 28
d 96, 144 **e** 16, 48 **f** 15, 21, 45

Homework sheet
Prime and composite numbers
MAT07NAHS10003

3-12 Lowest common multiple

Homework sheet
Whole numbers review
MAT07NAHS10004

- The multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, …
- The multiples of 7 are 7, 14, 21, 28, 35, 42, 49, 56, …

The **multiples** of a number are found by multiplying that number by the whole numbers 1, 2, 3, 4, ….

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Summary

The **lowest common multiple** (LCM) of two (or more) numbers is the smallest number that is a multiple of **both** (**or all**) of these numbers.

also called the **least** common multiple

Example 22

Find the lowest common multiple (LCM) of:

a 3 and 4 **b** 6 and 9

Solution

a Multiples of 4 are 4, 8, **12**, 16, 20, **24**, 28, 32, …
Multiples of 3 are 3, 6, 9, **12**, 15, 18, 21, **24**, …
The common multiples are **12**, **24**, …
The lowest common multiple of 3 and 4 is **12**.

b Multiples of 6 are 6, 12, **18**, 24, 30, **36**, 42, 48, **54**, …
Multiples of 9 are 9, **18**, 27, **36**, 45, **54**, 63, 72, 81, …
The lowest common multiple of 6 and 9 is **18**.

Using prime factors to find the LCM

Example 23

Use factor trees to find the LCM of 18 and 30.

Solution

- Draw factor trees for 18 and 30.

- Circle common prime factors: 3 and 2.
- Cross out one of the 3s and one of the 2s.
- Multiply the remaining factors to calculate the LCM.
- LCM of 18 and 30 $= 3 \times 3 \times 2 \times 5$
$= 90$

Summary

To find the lowest common multiple of two numbers using their prime factors:

1. Circle common prime factors
2. Cross out **one** of each pair of common factors
3. Multiply the remaining factors

Exercise 3-12 Lowest common multiple

Extra questions
Special numbers
MAT07NAEQ00014

1 Write the first 5 multiples of 7.

2 List the multiples of 4 between 13 and 45.

3 How many multiples of 13 are less than 220?

See Example 19

4 Find the lowest common multiple of each set of numbers.

a 3, 5	**b** 6, 7	**c** 4, 6	**d** 15, 10
e 5, 8	**f** 4, 10	**g** 10, 5	**h** 2, 8
i 9, 6	**j** 3, 7	**i** 3, 4, 5	**j** 4, 12, 10

5 Why are some lowest common multiples easier to find? Write down your strategy for finding LCMs.

6 On a spreadsheet, the lowest common multiple can be found using the function =**lcm()**.

a Enter =**lcm(8, 12)** into a spreadsheet cell to find the LCM of 8 and 12.

b Check your answers to question **4** using a spreadsheet.

See Example 20

7 Use factor trees to find the lowest common multiple of each pair of numbers.

a 28, 15 **b** 16, 25 **c** 44, 20 **d** 18, 60

Technology Lowest common multiple

This activity searches for the lowest common multiple of 8, 12 and 16 by listing their multiples first. Start a new spreadsheet and enter the information shown below.

	A	B	C	D
1	Number:	8	12	16
2	1			
3	2			
4	3			
5	4			
6	5			
7	6			
8	7			
9	8			
10	9			
11	10			
12	11			
13	12			
14	13			
15	14			
16	15			

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1 In cell B2, enter the formula =**\$B\$1*A2**. Use **Fill Down** to find the first 12 multiples of 8.
2 In cells C2 and D2, enter similar formulas and **Fill Down** to find the first 12 multiples of 12 and 16. [Hint: Only change the absolute cell reference.]
3 By examining the columns of multiples, find the lowest common multiple of 8, 12 and 16.
4 By using the formula =**lcm(8, 12, 16)**, find the lowest common multiple of 8, 12 and 16.
5 Modify your spreadsheet to find the LCM of each of the following sets of numbers. Note: You may need to extend beyond the first 12 multiples.
  a 6 and 15 b 12 and 18 c 3, 7 and 15 d 48, 60 and 75

Power plus

1 For each set of numbers, evaluate and write in ascending order.
  a $2^3, 3^2, 3^5, 5^3, 2^5, 5^2$ b $4^4, 7^3, 3^5, 8^2, 5^2, 6^3$ c $100^2, 11^4, 2^7, 3^4, 5^3$
2 Evaluate each square number.
  a 1^2 b 11^2 c 111^2 d 1111^2
3 Based on the patterns in your answers to question **2**, evaluate each square number.
  a $11\,111^2$ b $1\,111\,111^2$ c $111\,111\,111^2$ d $1\,111.1111^2$
4 Find the square root of:
  a $3 \times 3 \times 2 \times 2$ b $5 \times 5 \times 4 \times 4 \times 3 \times 3$
5 Evaluate each of the following without using a calculator.
  a $\sqrt{9^2}$ b $\sqrt{3^4}$ c $\sqrt{5^4 \times 2^6}$
  d $\sqrt[3]{2^9}$ e $\sqrt[3]{3^3 \times 5^6}$ f $\sqrt[4]{16}$
6 Find the square root of each number without using a calculator.
  a 2500 b 8100 c 10 000
  d 1 000 000 e 1 210 000 f 100 000 000
  g 640 000 h 176 400 i 10 000 000 000
7 Two prime numbers that differ by 2 are called twin primes. For example, 11 and 13 are twin primes, but 23 and 29 are not. Find all 8 pairs of twin primes below 100.

Chapter 3 review

Puzzle sheet
Number find-a-word
MAT07NAPS10012

Language of maths

composite	factor	lowest common multiple (LCM)	prime factor
cube	factor tree	multiple	round down
divisibility test	greatest common divisor (GCD)	power	round up
divisor	index notation	prime	square
estimate			square root

Quiz
Whole numbers
MAT07NAQZ00002

1 What is another name for:

a divisor? **b** highest common factor?

2 What is used as an abbreviation for repeated multiplication?

3 What type of numbers have more than 2 factors?

4 Describe the divisibility test for 3.

5 Find a non-mathematical meaning of:

a prime **b** index

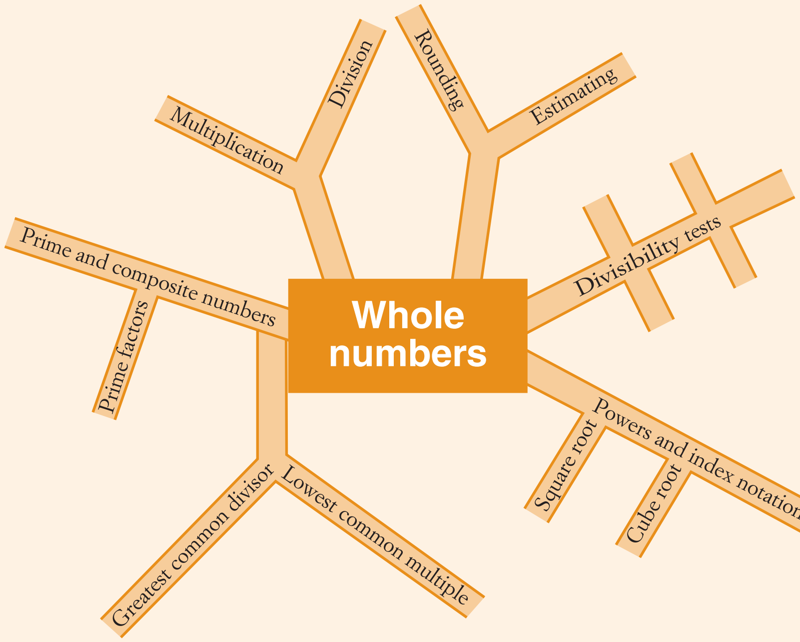
6 What helps us to write a number as a product of its prime factors?

Topic overview

Worksheet
Mind map: Whole numbers
MAT07NAWK10025

- Write in your own words what you have learnt about whole numbers.
- What was your favourite part of this topic?
- What parts of this topic didn't you understand? Talk to your teacher or a friend about them.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



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Chapter 3 revision

1 Round 87 531 to the nearest: *See* Exercise 3-01
a thousand **b** hundred **c** ten **d** ten thousand

2 Estimate the value of each expression. *See* Exercise 3-01
a $425 + 81 + 940 + 22$ **b** $864 + 250 - 417$
c 93×19 **d** $2402 \div 32$

3 Evaluate each product. *See* Exercise 3-02
a 12×4 **b** 34×5 **c** 33×9
d 25×8 **e** 126×7 **f** 565×18

4 Evaluate each quotient. *See* Exercise 3-03
a $2400 \div 10$ **b** $620 \div 5$ **c** $1296 \div 8$
d $528 \div 4$ **e** $144 \div 9$ **f** $847 \div 7$

5 Test whether 1208 is divisible by: *See* Exercise 3-04
a 2 **b** 3 **c** 4 **d** 5
e 6 **f** 8 **g** 9 **h** 10

6 Which number is divisible by both 3 and 8? Select the correct answer **A**, **B**, **C** or **D**. *See* Exercise 3-04
A 30 **B** 24 **C** 90 **D** 94

7 Evaluate each quotient. *See* Exercise 3-05
a $3079 \div 12$ **b** $650 \div 25$ **c** $2976 \div 24$

8 Write each number using Roman numerals. *See* Exercise 3-06
a 42 **b** 38 **c** 89 **d** 347

9 Write each expression using index notation. *See* Exercise 3-07
a $3 \times 3 \times 3 \times 3$ **b** $6 \times 6 \times 6 \times 6 \times 6$
c $11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11$

10 Evaluate each power.
a 6^2 **b** 5^3 **c** 10^5 **d** 2^8

11 Evaluate each root. *See* Exercise 3-08
a $\sqrt{49}$ **b** $\sqrt{144}$ **c** $\sqrt[3]{8000}$ **d** $\sqrt[3]{729}$

12 **a** List the prime numbers between 26 and 50. *See* Exercise 3-09
b List the composite numbers between 45 and 70.

13 Use factor trees to express each number as a product of its prime factors. *See* Exercise 3-10
a 60 **b** 200 **c** 144

14 Use factor trees to evaluate each root. *See* Exercise 3-10
a $\sqrt{256}$ **b** $\sqrt[3]{5832}$

15 Find the greatest common divisor of each pair of numbers. *See* Exercise 3-11
a 20 and 48 **b** 36 and 84

16 Use factor trees to find the greatest common divisor of 243 and 162. *See* Exercise 3-11

17 Find the lowest common multiple of each pair of numbers. *See* Exercise 3-12
a 6 and 8 **b** 18 and 10

18 Use factor trees to find the lowest common multiple of 25 and 16. *See* Exercise 3-12