Giving meaning to Multiplication & Division of fractions

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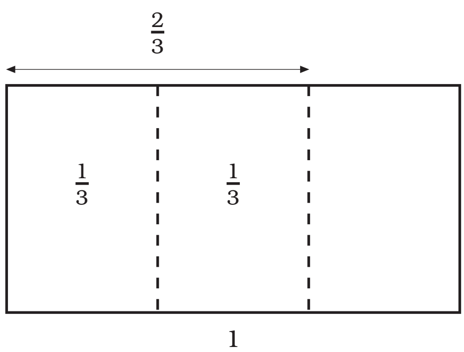
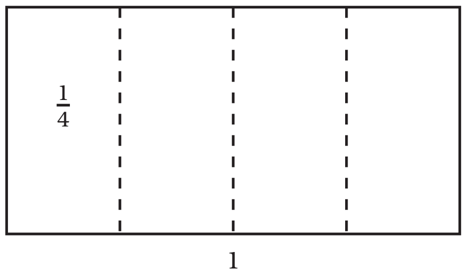
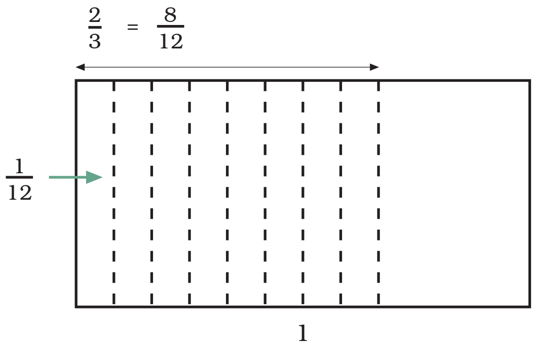
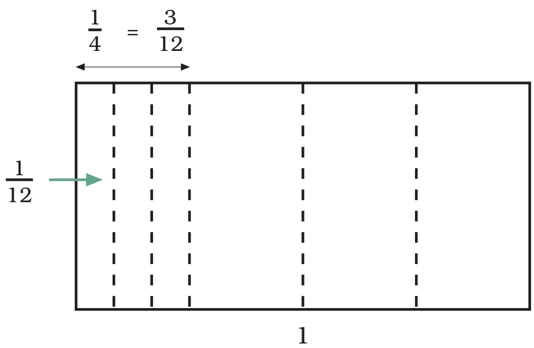
We have some experience with addition and subtraction of fractions using area models like rectangles and circles ('pie charts') or linear models like the number line and fraction strips ('fraction bars'). These models compel the user to come up with a common denominator in order to add (or subtract) two fractions. For example, suppose we wish to perform the operation $\frac{2}{3} + \frac{1}{4}$. Using fraction strips we would compose $\frac{2}{3}$ and $\frac{1}{4}$:



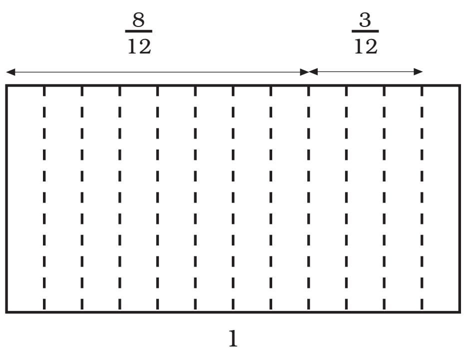


We would then reform the thirds into four equal parts (creating twelfths) and the fourth into three equal parts (also creating twelfths, the common denominator).



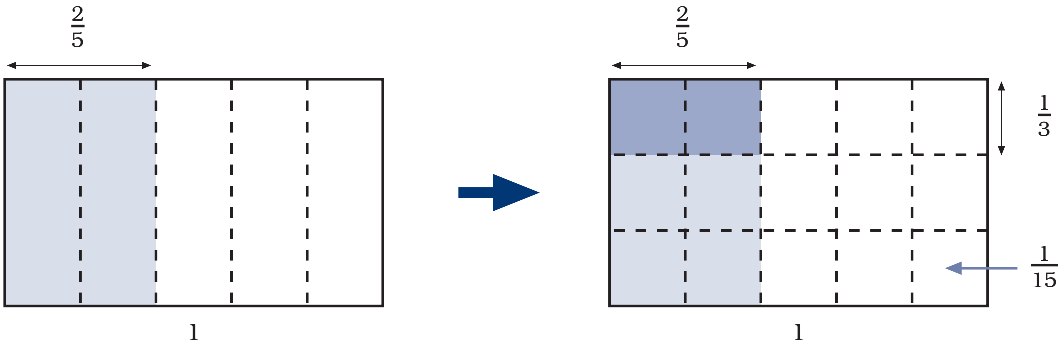


We can now combine the two (either by cutting the parts off with scissors or overlapping them), completing the 'computation' as $\frac{8}{12} + \frac{3}{12} = \frac{11}{12}$.



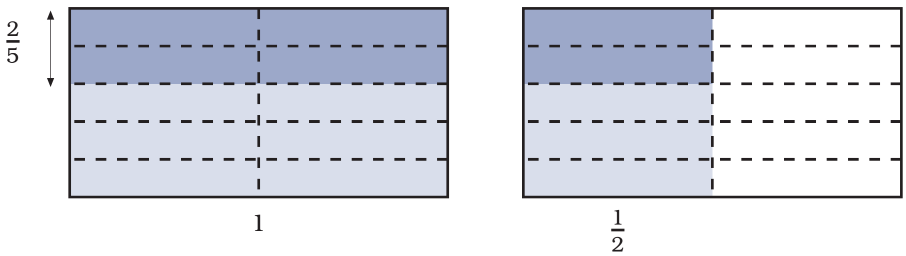
When multiplying fractions we do not teach students to find a common denominator and the algorithm is so simple it begs for understanding. By using similar models we can encourage students to think about the process of fraction multiplication.

The best interpretation of fraction multiplication is that of a part *of* something. Thus $\frac{1}{3} \times \frac{2}{5}$ is thought of as $\frac{1}{3}$ of $\frac{2}{5}$. This has a comforting interpretation steeped in experience and real world application. We can easily envision getting 'a third' of a candy bar or thinking of a third of the class being selected for a project. So it is when we seek an area model for $\frac{1}{3}$ of $\frac{2}{5}$ we first draw the two fifths, then take a third of it.

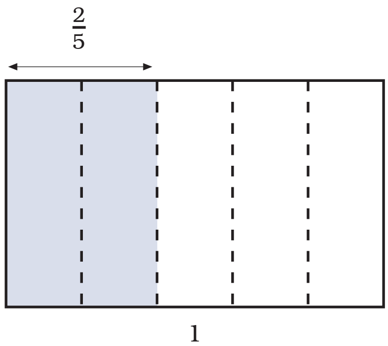


We now interpret the answer from the figure (without relying on the standard algorithm). Since the unit rectangle (area of 1) is partitioned vertically into fifths, and horizontally into thirds, the small rectangles formed by the intersections of the vertical and horizontal lines are fifteenths (since $5 \times 3 = 15$ are formed). Thus $\frac{1}{3}$ of $\frac{2}{5} = \frac{2}{15}$ (shaded area).

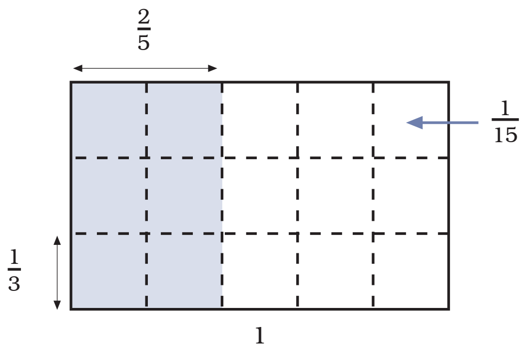
Here is another example: $\frac{2}{5} \times 1\frac{1}{2}$, or $\frac{2}{5}$ of $1\frac{1}{2}$.



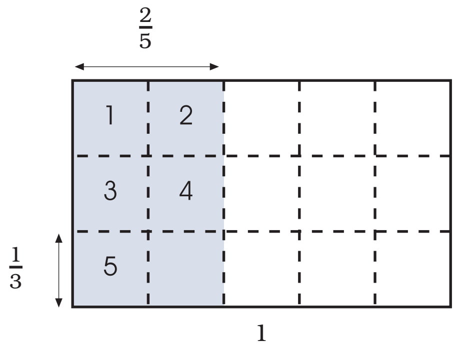
The model for division is a bit more challenging. For the first example, let us try $\frac{2}{5} \div \frac{1}{3}$. Since we are asking, 'how many one-thirds are in two-fifths,' we begin by drawing $\frac{2}{5}$ on the unit rectangle.



We need to figure out a way to make one-thirds out of the area now marked as $\frac{2}{5}$. This requires a similar unit of area or 'common denominator' that can be used to compare. This is accomplished by drawing horizontal lines for the 'thirds'.

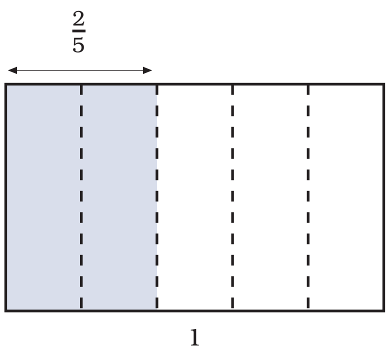


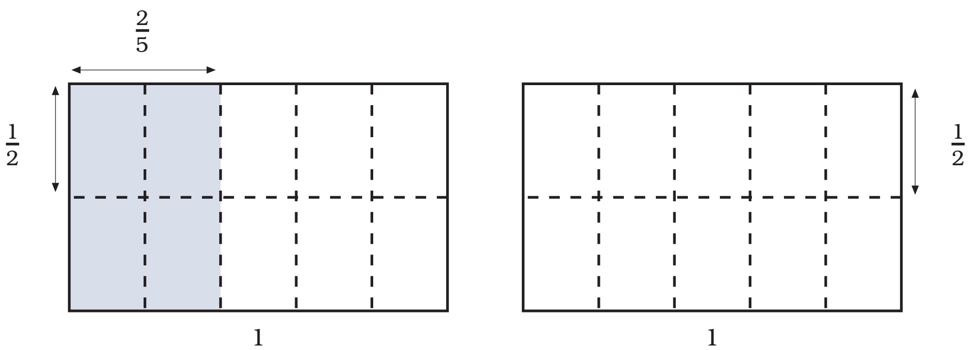
Now a third is made up of 5 small rectangles so, in the area marked $\frac{2}{5}$ we can make *one* of the one-thirds by counting off five small rectangles.



Since it takes 5 small rectangles to make $\frac{1}{3}$, and we have 1 small rectangle left over, there is $1\frac{1}{5}$ one-thirds in $\frac{2}{5}$. Therefore, $\frac{2}{5} \div \frac{1}{3} = 1\frac{1}{5}$. This model gives one a good 'feel' for the concept of division of fractions and does not invoke the standard algorithm. That can come later after the conceptual understanding is in place: $\frac{2}{5} \div \frac{1}{3} = \frac{2}{5} \times \frac{3}{1} = \frac{6}{5} = 1\frac{1}{5}$.

Let us try one last example: $\frac{2}{5} \div 1\frac{1}{2}$.





Since it takes 15 small rectangles to make $1\frac{1}{2}$, and we only have 4 small rectangles in $\frac{2}{5}$, we have 4 of the 15 required so $\frac{2}{5} \div 1\frac{1}{2} = \frac{4}{15}$.

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